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Gearbox Project

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Calculate Shaft Diameters (Similar to Case Study 7B)

Analytical Solution

Given Data:

d_p, diameter of pinion's pitch circle (in)
d_p = 4

r_g, radius of the gear's pitch circle (in).
r_g = 5

T_max, maximum Torque (lb*in)
T_max = 800

T_min, minimum Torque (lb*in)
T_min = -250

Sut, ultamate tensile strength (psi)
Sut = 64000

Sut, yield strength (kpsi)
Sy = 54

Nsf, safety factor
Nsf = 3

PA, pressure angle (degrees)
PA = 20

The type of load applied to both cylinders is bending i.e. no axial forces.

The output shaft has a circular cross section with a diameter of 1.1423 in. The cylinder is obviously in rotation.

The input shaft has a circular cross section with a diameter of 0.8686 in. The cylinder is obviously in rotation

Both cylinders have machined surfaces, are operated at about 150 degrees F, have a reliability of 50 percent, and are made of steel.

The geometric stress concentration factor is assumed to be 3.

<u>Solution:</u>

1. Calculation of Maximum/Minimum Tangential Forces (Ft):

$Ft\_max = T\_max / r\_g$
$Ft\_max = 160$

$Ft\_min = T\_min / r\_g$
$Ft\_min = -50$

2. Calculation of Total Maximum/Minimum Forces (F):

$F\_max = Ft\_max / \cos(Phi)$
$F\_max = 170.2684$

$F\_min = Ft\_min / \cos(Phi)$
$F\_min = -53.2089$

3. Calculation of Maximum/Minimum Moments (M):

$M\_max = F\_max * (d\_p/4)$
$M\_max = 170.2684$

$M\_min = F\_min * (d\_p/4)$
$M\_min = -53.2089$

4. Calculation of Mean/Amplitude Moments:

$Mm = (1/2) * (M\_max + M\_min)$
$Mm = 58.5298$

$Ma = (1/2) * (M\_max - M\_min)$
$Ma = 111.7387$

5. Calculation of Mean/Amplitude Torques:

Tm = (1/2) * (T_max + T_min)
Tm = 275

Ta = (1/2) * (T_max - T_min)
Ta = 525

6. C factores and strength of output shaft:

C_load = 1, Because no axial forces are present.

A_95 = 0.0766.*(1.1423).^2
d_equi = (A_95/0.0766)^(1/2)
C_size = 0.869.*d_equi.^(-0.097) = 0.8579.

C_surf = 2.7.*(S_ut).^(-0.265) = 0.8969
C_temp = 1, Because T < 840 degrees F.
C_reliab = 1, Because reliability equals 50%.

S_prime = 0.5.*S_ut = 32.

7. C factores and strength of input shaft:

C_load = 1, Because no axial forces are present.

A_95 = 0.0766.*(0.8686).^2
d_equi = (A_95/0.0766)^(1/2)
C_size = 0.869.*d_equi.^(-0.097) = 0.8810

C_surf = 2.7.*(S_ut).^(-0.265) = 0.8969
C_temp = 1, Because T < 840 degrees F.
C_reliab = 1, Because reliability equals 50%.

S_prime = 0.5.*S_ut = 32

8. Corrected strengths:

Se_i, Corrected strength for input shaft (psi)
Se_i = C_load*C_size*C_surf*C_temp*C_reliab*S_prime = 2.5283e+004

Se_o, Corrected strength for output shaft (psi)
Se_o = C_load*C_size*C_surf*C_temp*C_reliab*S_prime = 2.4620e+004

9. Notch Sensitivities

r, notch radius – assumed value.
r = 0.0100

a_b, Neubers constant for bending stress (in).
a_b = 0.0100

q_b, Notch Sensitivity for bending stress in keyway.
q_b = 1 / (1 + sqrt(a_b/r)) = 0.5000

a_t, Neubers constant for torsion stress (in).
a_t = 0.0056

q_t, Notch Sensitivity for torsion stress in keyway.
q_t = 1 / (1 + sqrt(a_t/r)) = 0.5714

10. Fatigue Stress Concentration Factors:

Kf, Fatigue Stress Consentration Factors for bending stress in keyway.
Kf = 1 + q_b*(Kt - 1) = 2

Kfs, Fatigue Stress Consentration Factors for tortion stress in keyway.
Kfs = 1 + q_t*(Kt - 1) = 2.1429

Kfm = Kf
Kfsm = Kfs

11. Diameter Calculations:

d_output, output diameter (in)

d_output = (((32.*Nsf)./pi).*((sqrt((Kf.*Ma).^2 + (3/4).*(Kfs.*Ta_in).^2)./Se_i) +

(sqrt((Kfm.*Mm).^2 + (3/4).*(Kfsm.*Tm_in).^2)./Sut))).^(1/3) = 1.1423

Tm_in, mean tourque for inner shaft (lbf*in)
Tm_in = 0.4*Tm = 110

Ta_in, alternating tourque for inner shaft (lbf*in)
Ta_in = 0.4*Ta = 210

d_input, input diameter (in)

d_input = (((32.*Nsf)./pi).*((sqrt((Kf.*Ma).^2 + (3/4).*(Kfs.*Ta_in).^2)./Se_i) +

(sqrt((Kfm.*Mm).^2 + (3/4).*(Kfsm.*Tm_in).^2)./Sut))).^(1/3) = 0.8686

Choose a standardized sizes of d_output = 1.15 in and d_input = 0.87 in. Therefore: r_output = 0.575 in and r_input = 0.435 in.

Find Gear Properties and Safety Factors (Similar to Case Study 7C)

Analytical Solution

Given Data:

m_G, gear ratio (Ng/Np).
m_G = 2.5

omega_g, angular velocity of gear (rad/s).
omega_g = 157.0796

PA, pressure angle (degrees).
PA = 20

x_p, elongation (percent).
x_p = 0

J_p, Bending strength geometry factor for pinion (Table 11-9).
J_p = 0.34

J_g, Bending strength geometry factor for gear (Table 11-9).
J_g = 0.4000

T_max, maximum Torque (lb*in).
T_max = 585

T_min, minimum Torque (lb*in).
T_min = -175

E, Modulus of elastisity (psi).
E = 30000000

v, Poissons ratio.
v = 0.28

Both gears have a reliability of 99%, they are both non-idle, have a part life of 4.6800e+009 cycles, have size dimensions of 1, have a gear quality (Qv) of 10, and are operated at about 150 degrees F. Both gears have Ks values of 1.

HB_p, Brinell Hardness for pinion.
HB_p = 250

HB_g, Brinell Hardness for gear.
HB_g = 250

N_p, number of teeth on pinion.
N_p = 20

d_p, diameter of pinion (in).
d_p = 4

d_g, diameter of gear (in).
d_g = 10

<u>Solution:</u>

1. Calculation of diametric pitch:

N_g, Number of teeth on gear.
N_g, N_g = N_p.*m_G = 50

p_d, diametric pitch.
p_d = N_p./d_p = 5

2. Calculation of face width:

FW, face width (in) - Assume nominal value of 12.
FW = 12/p_d = 2.4000

3. Calculation of Tangential load:

Wt_max, Maximum Tangential force (lbf).
Wt_max = T_max./(d_g./2) = 117

Wt_min, Minimum Tangential force (lbf).
Wt_min = T_min./(d_g./2) = -35

4. Calculation of Tooth velocity:

Vt, Tooth velocity (ft/min).
Vt = 4.1888+003

5. K values and Bending Stress for pinion:

Ka = 2, Assumed value of two.

FW > 2 so use the equation below:
Km = ((1.8-1.7)/(9-6)).*((FW)-6) + 1.7 = 1.5800

Because Qv >= 6 and Qv <= 11 can use the equation below:
B = (12 - Qv).^(2/3)./4
A = 50 + 56.*(1 - B)
Kv = (A/(A + (Vt).^(1/2))).^B = 0.7968

Ks = 1, Assumed value.
Kb = 1, Because the gears are solid.
Ki = 1, Because the gear and pinion are non-idle

Sigma_bending_p = (Wt*p_d)/((FW)*J)*(Ka*Km)/(Kv)*Ks*Kb*Ki = 2.8432e+003

6. K values and Bending Stress for gear:

Ka = 2, Assumed value of two.

FW > 2 so use the equation below:
Km = ((1.8-1.7)/(9-6)).*((FW)-6) + 1.7 = 1.5800

Because Qv >= 6 and Qv <= 11 can use the equation below:
B = (12 - Qv).^(2/3)./4
A = 50 + 56.*(1 - B)
Kv = (A/(A + (Vt).^(1/2))).^B = 0.7968

Ks = 1, Assumed value.
Kb = 1, Because the gears are solid.
Ki = 1, Because the gear and pinion are non-idle

Sigma_bending_g = (Wt*p_d)/((FW)*J)*(Ka*Km)/(Kv)*Ks*Kb*Ki = 2.4167e+003

7. Surface Stresses:

Dis_pg, Distance between center of gears.
Dis_pg = (d_p + d_g)./2 = 7

I_pg, Surface Geometry Factor for pinion gear pair.
I_pg = cos(phi)./((1/rho_p + 1/rho_g).*d_p) = 0.0999

Cp, Elastic coefficient.
Cp = sqrt(1./(pi.*(((1 - v.^2)./(E)) + ((1 - v.^2)./(E))))) = 2.2761e+003

8. C values and Surface Stresses:

Because Qv >= 6 and Qv <= 11 can use the equation below:
B = (12 - Qv).^(2/3)./4
A = 50 + 56.*(1 - B)
Kv = (A/(A + (Vt).^(1/2))).^B = 0.7968
Cv = Kv

FW > 2 so use the equation below:
Km = ((1.8-1.7)/(9-6)).*((FW)-6) + 1.7 = 1.5800
Cm = Km

Ca = 2, Assumed value of two.

Ks = 1, Assumed value.
Cs = Ks

Cf = 1, Assumed surface factor

Sigma_surface_pg = 5.0058e+004

9. Fatigue Bending Strength and K values:

S_fb_prime = -274 + 167.*HB_p - .152.*HB_p.^2 = 31976 psi

Kl = 1.3558.*(N).^(-0.0178) = 0.9121
Kt = 1, Because temperature is less than 250 degrees F.
Kr = 1, Because reliability is 99%.

10. Corrected Fatigue Bending Strength:

S_fb = Kl./(Kt.*Kr).*S_fb_prime = 2.9167e+004 psi

11. Fatigue Surface Strength and K values:

Ct = 1, Because temperature is less than 250 degrees F.

Cr = 1, Because reliability is 99%.

Cl = 1.4488*(N)^(-0.023) = 0.8681

m_G = N_g/N_p
Ch = Ch = 1 + A*(m_G -1) = 1

12. Corrected Fatigue Surface Strength:

S_fs = (Cl*Ch)/(Ct*Cr)*S_fs_prime = 9.3543e+004

13. Safety Factors:

SF_bend_pinion, Safty Factor for pinion in bending.
SF_bend_pinion = S_fb./Sigma_bending_p = 10.2584

SF_bend_gear, Safty Factor for gear in bending.
SF_bend_gear = S_fb./Sigma_bending_g = 12.0687

SF_surf_pg, Safty Factor for the gear and pinion surfaces.
SF_surf_pg = (S_fs./Sigma_surface_pg).^2 = 3.4920

Verification of results in SolidWorks and Nastran4D.

To increase the accuracy of COSMOS a material was created that matched the theoretical properties. In this case the property values of steel AISI 304 were altered to match the theoretical ones used. Note that Poisson's ratio, Yield strength, and Tensile strength values given in table #1 match the theoretical values exactly.

Table #1 Properties of the Material.

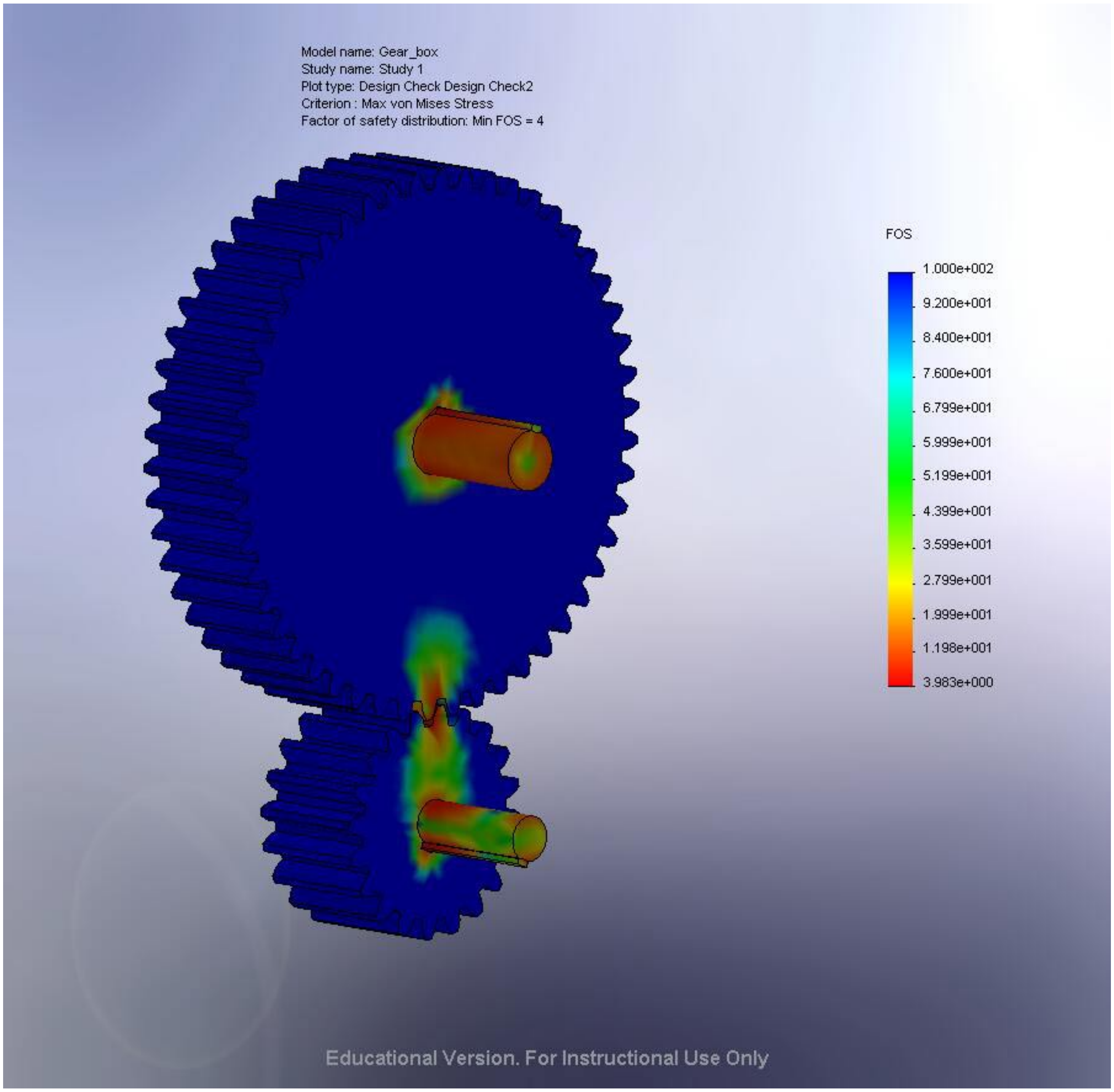
Property Name	Value	Units	Value Type
Elastic modulus	2.7557e+007	psi	Constant
Poisson's ratio	0.28	NA	Constant
Shear modulus	1.0878e+007	psi	Constant
Mass density	0.28902	lb/in^3	Constant
Tensile strength	64000	psi	Constant
Yield strength	54000	psi	Constant
Thermal expansion coefficient	1e-005	/Fahrenheit	Constant
Thermal conductivity	0.000214	BTU/(in.s.F)	Constant
Specific heat	0.11945	Btu/(lb.F)	Constant

Study Results

Table #2 Resulting Stress, Strain, and Displacement.

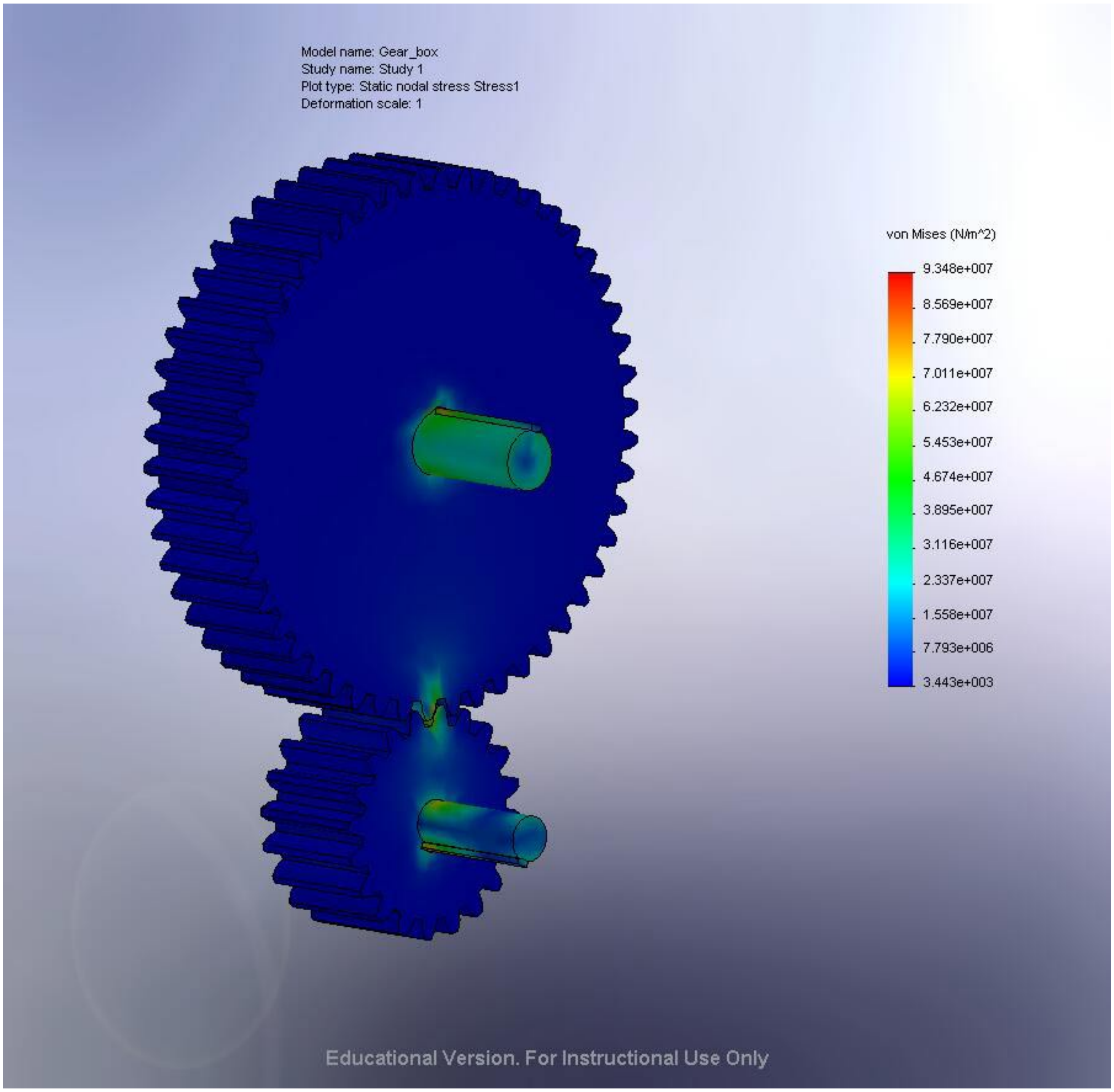
Name	Type	Min	Location	Max	Location
Stress1	VON: von Mises Stress	3443.24 N/m^2 Node: 10793	(-1.8 in, 10.5678 in, 4.74078 in)	9.34776e+007 N/m^2 Node: 16985	(-2.6 in, -0.132321 in, -2.14393 in)
Displacement1	URES: Resultant Displacement	0 m Node: 16464	(-5 in, 0.720108 in, -1.80261 in)	0.000371493 m Node: 1009	(0 in, 10.4128 in, 5.26186 in)
Strain1	ESTRN: Equivalent Strain	1.54644e-008 Node: 10793	(-1.8 in, 10.5678 in, 4.74078 in)	0.000419829 Node: 16985	(-2.6 in, -0.132321 in, -2.14393 in)

For COSMOS to analyze the gear box a torque was applied to the gears shaft (output shaft) and a restraint was applied to the pinion shaft (input shaft). The applied torque was 800 lb*in. Ideally the torque would have been applied to one end the output shaft, and the restraint would be placed on the opposite end of the input shaft. But because not ball bearings are being used to restrain the shafts in place this results in an unstable unsolvable system.



Gear_box-Study 1-Design Check-Design Check2

The factor of safety calculated in COSMOS is 4 which is very close to the theoretical value of 3. A possible reason for this inaccuracy is that for COSMOS to run an analysis a mesh bust be used and this mesh treats the cylinders and the gears they are attached to as one solid part. This would lead to inflated values of the safety factor which is the case.



Gear_box-Study 1-Stress-Stress1