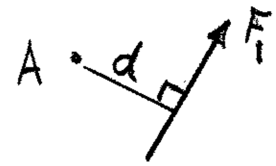
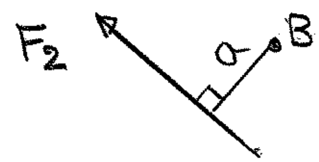
MOMENT ABOUT A POINT

MOMENT ABOUT A POINT = FORCE TIMES PERPENDICULAR DISTANCE FROM THE POINT TO THE FORCE

2 SIGN CONVENTION – COUNTERCLOCKWISE MOMENT IS POSITIVE (RHR).

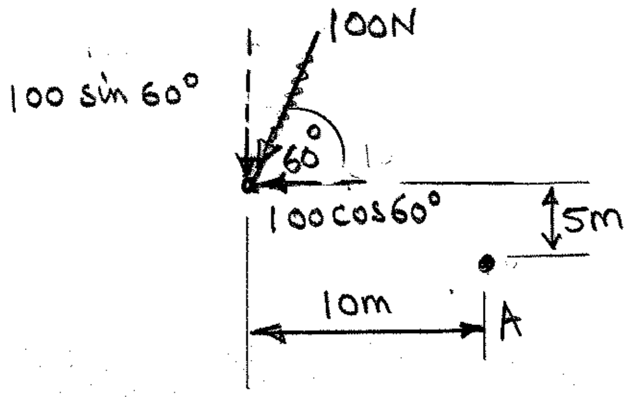


$M_A = F_1 d$
(counterclockwise)



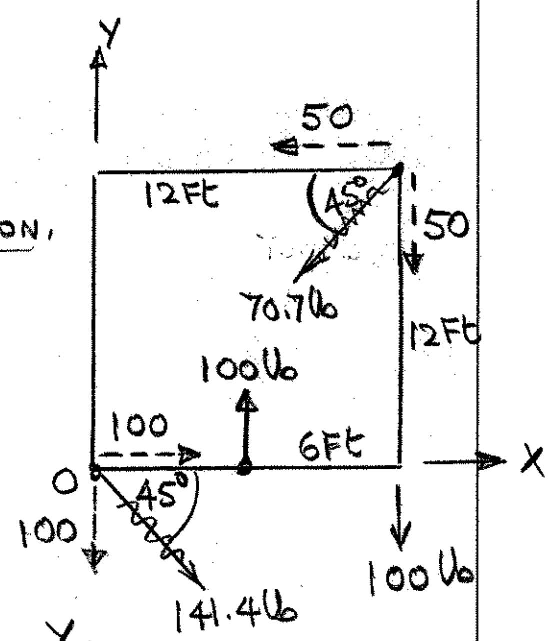
$M_B = -Fa$ (clockwise)

3 INCLINED FORCES : RESOLVE FORCE INTO X- AND Y- COMPONENTS FIRST



$M_A = (100 \sin 60^\circ)10 + (100 \cos 60^\circ)(5)$

4 MOMENT CAUSED BY SEVERAL FORCES
FIND RESULTANT FORCE AND POINT OF ACTION.



RESOLVE INTO X- AND Y- COMPONENTS

$\Sigma F_x = -50 + 100 = 50\text{ lb}$

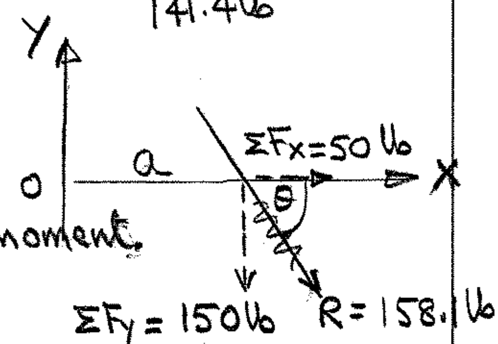
$\Sigma F_y = -50 + 100 - 100 - 100 = -150\text{ lb}$

$R^2 = 50^2 + 150^2 \qquad R = 158.1\text{ lb}$

$M_O = 50(12) - 50(12) + 100(6) - 100(12)$

$M_O = -600\text{ lb-ft.}$

Let R pass through a point distance 'a' from O. Only vertical component causes moment.

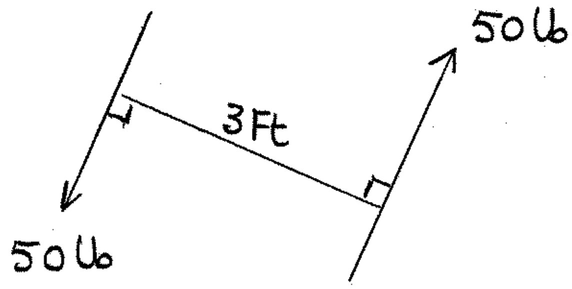


EQUIVALENT MOMENT: $-600 = -150(a)$

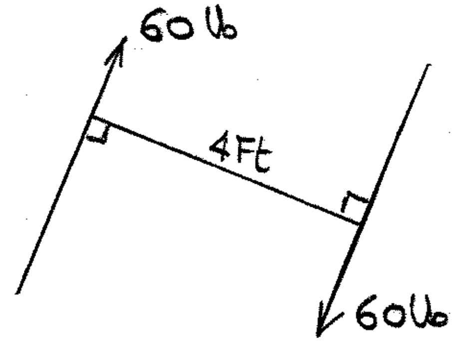
$a = 4\text{ Ft}$

$\tan\theta = \frac{-150}{50} = -3 \; ; \quad \theta = -71.57^\circ$ OR 288.43°

5 COUPLE CAUSED BY EQUAL AND OPPOSITE FORCES.

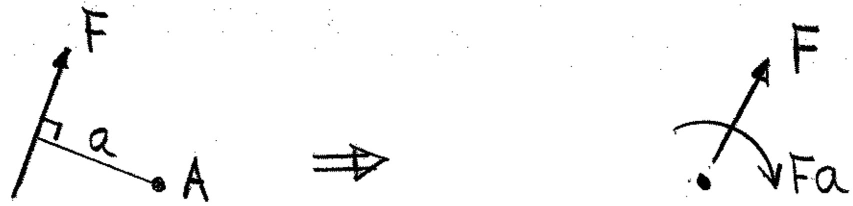


$M = 50(3) = 150$ ft-lb



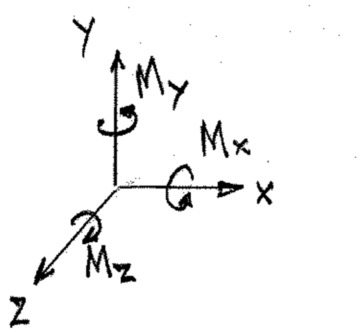
$M = -60(4) = -240$ ft-lb.

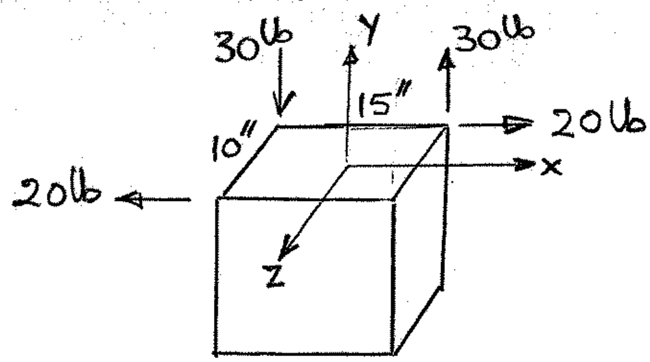
6. LATERAL DISPLACEMENT OF A FORCE = FORCE PLUS MOMENT



RIGHT HAND RULE - RHR

GRAB AXIS WITH RIGHT HAND SUCH THAT FINGERS CURL IN SAME DIRECTION AS MOMENT. THUMB GIVES DIRECTION OF VECTOR FOR MOMENT



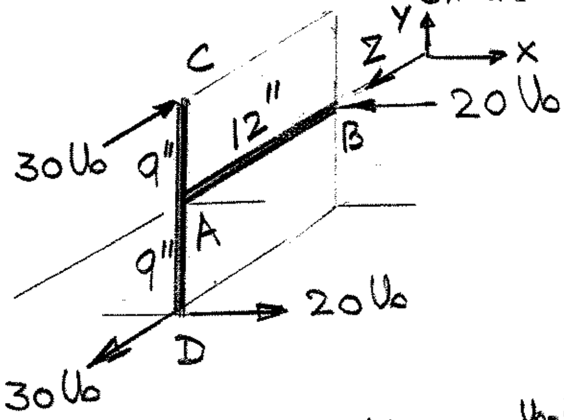


M_X IS POSITIVE
M_Y IS POSITIVE
M_Z IS NEGATIVE

Two 20-lb create $M_Y = -20(10) = -200$ lb-in

Two 30-lb $\longrightarrow$ $M_Z = +30(15) = 450$ lb-

EXAMPLE ON COUPLES: DETERMINE COMPONENTS OF EQUIVALENT SINGLE COUPLE.



Two 30-lb create $M_x = -30(18)$

$M_x = -540$ lb-in

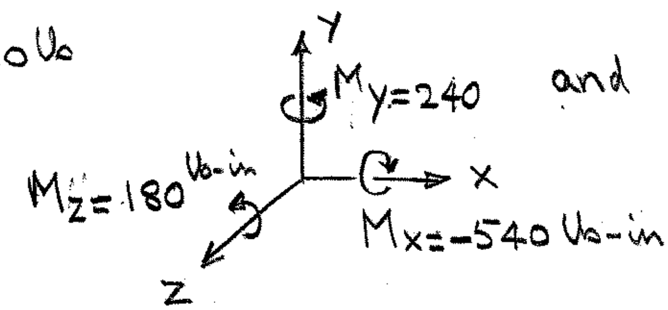
NEGATIVE SIGN BY RHR.

Two 20-lb create $M_z = +20(9)$

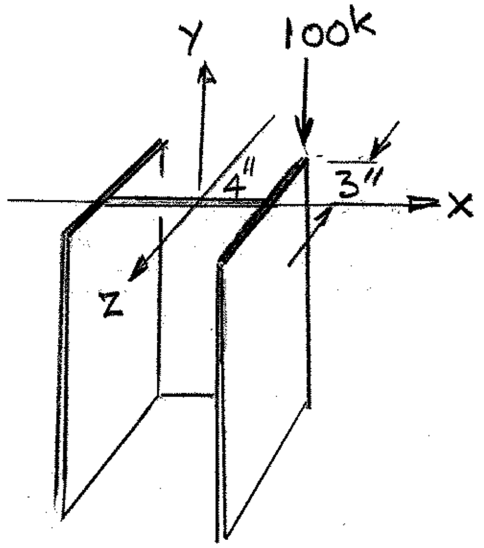
$M_z = +180$ lb-in

and $M_y = +20(12)$

$M_y = +240$ lb-in

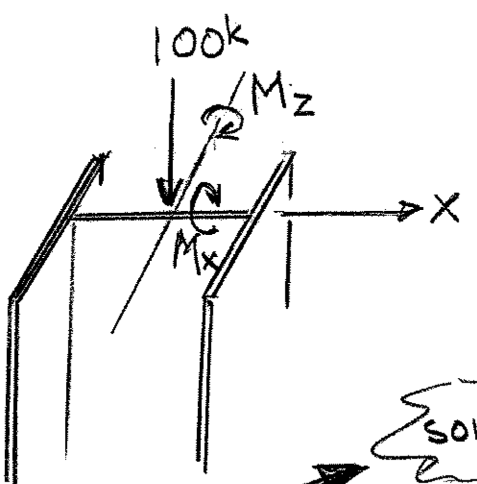


$$\underset{\sim}{M} = -540\,\underset{\sim}{i} + 240\,\underset{\sim}{j} + 180\,\underset{\sim}{k}$$



COLUMN IS SUBJECTED TO AN ECCENTRIC LOAD OF 100^k (NOT PASSING THROUGH CENTROID).

TRANSFER LOAD TO CENTROID.



IN ADDITION TO AXIAL FORCE OF 100^k, THERE EXIST MOMENTS ABOUT X- AND Z-AXIS WHICH WILL CAUSE BENDING

$M_x = -100(3) = -300$ k-in. RHR

$M_z = -100(4) = -400$ k-in.

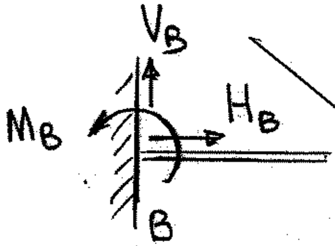
SOLVE PROBLEMS: 3.55, 3.56, 3.58, 3.62, 368, 370
3.80, 382

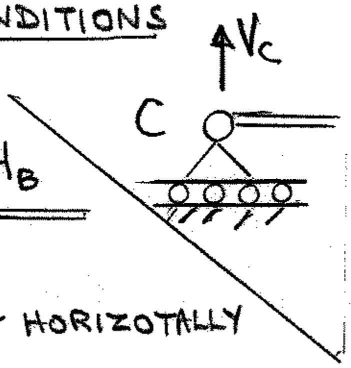
EQUILIBRIUM – NONCONCURRENT FORCES

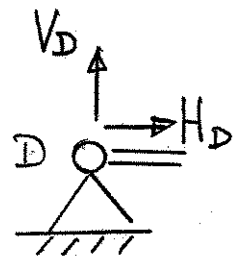
[2D] PROBLEMS : $\Sigma F_x = 0$; $\Sigma F_y = 0$; $\Sigma M = 0$

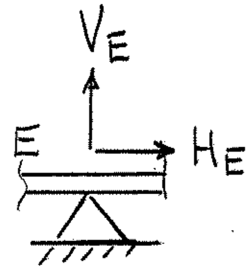
[3D] PROBLEMS : $\Sigma F_x = 0$; $\Sigma F_y = 0$; $\Sigma F_z = 0$; $\Sigma M_x = \Sigma M_y = \Sigma M_z = 0$

SUPPORT CONDITIONS





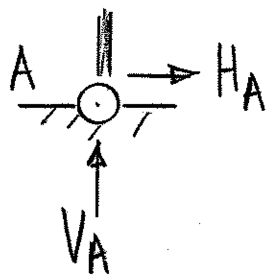


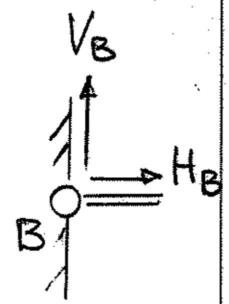


NO MOVEMENT HORIZOTALLY

NO MOVEMENT VERTICALLY

NO ROTATION





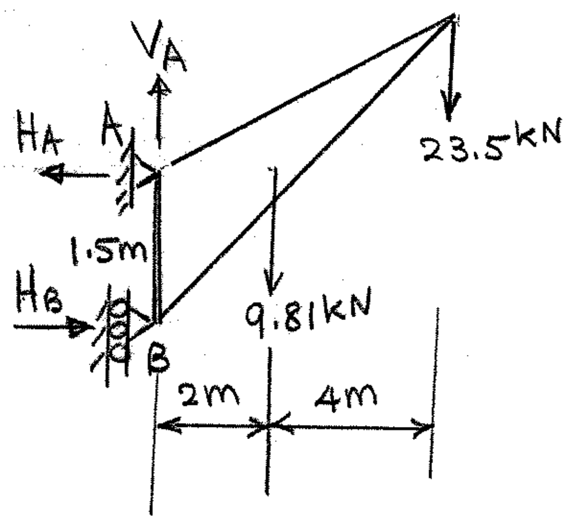
EXAMPLE

NOTE 3 REACTIONS.

ALWAYS APPLY $\Sigma M = 0$ FIRST

ABOUT POINT WHERE 2 REACTIONS

PASS.



$\Sigma M_A = 0$:

$+H_B(1.5) - 23.5(6) - 9.81(2) = 0$

$H_B = 107.1$ kN

$\Sigma F_x = 0$: $-H_A + H_B = 0$; $H_A = H_B = 107.1$ kN

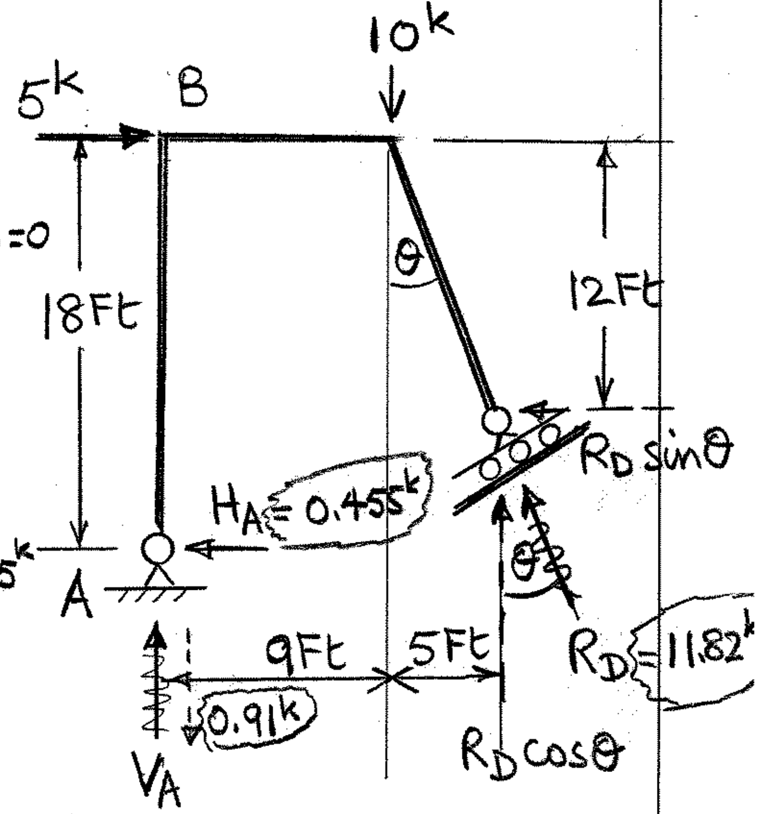
$\Sigma F_y = 0$: $V_A - 23.5 - 9.81 = 0$; $V_A = 33.31$ kN

EXAMPLE:

3 reactions. Assume V_A points upward. H_A to the left
R_D perpendicular to the plane. NOTE 5, 12, 13 TRIANGLE

~~ALWAYS~~ APPLY $\Sigma M = 0$ FIRST.

CHOOSE A MOST CONVENIENT POINT WHERE 2 REACTIONS PASS. IN THIS CASE – POINT 'A'. TAKE MOMENT ABOUT POINT 'A'. BUT RESOLVE R_D INTO X- AND Y- COMPONENTS BEFORE DOING CALCULATIONS.



$\Sigma M_A = 0$:

$-5(18) - 10(9) + R_D \sin\theta(6) + R_D \cos\theta(14) = 0$

$-90 - 90 + R_D\left(\frac{5}{13}\right)(6) + R_D\left(\frac{12}{13}\right)(14) = 0$

$R_D = 11.82^k$

$\Sigma F_x = 0: \; 5 - H_A - R_D \sin\theta = 0$

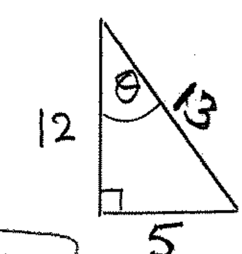
$H_A = 5 - 11.82\left(\frac{5}{13}\right) = 0.455^k$

$\Sigma F_y = 0: \; V_A + R_D \cos\theta - 10 = 0$

$V_A = 10 - R_D \cos\theta$

$V_A = 10 - 11.82\left(\frac{12}{13}\right) = -0.91^k$

ORIGINAL DIRECTION CHOSEN FOR V_A WAS WRONG.
CHANGE DIRECTION OF V_A.

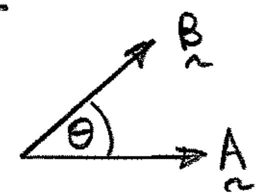


SOLVE PROBLEMS: 4.4, 4.10, 4.16, 4.18(c), 4.23, 4.30, 4.36

[3D] PROBLEMS

$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$ $\qquad A = \sqrt{A_x^2 + A_y^2 + A_z^2}$ MAGNITUDE

$\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$ $\qquad B = \sqrt{B_x^2 + B_y^2 + B_z^2}$



SCALAR PRODUCT: $\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A} = AB\cos\theta$

$\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z$

CAN USE ABOVE TO CALCULATE θ.

USE OF SCALAR PRODUCT — CALCULATE PROJECTION OF A VECTOR ON A LINE.

EXAMPLE: VECTOR $\mathbf{A} = 30\mathbf{i} + 50\mathbf{j} + 70\mathbf{k}$; LINE $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + 1\mathbf{k}$

FIRST FIND UNIT VECTOR $\mathbf{I}$ in the direction of $\mathbf{r}$.

Magnitude $r = (3^2 + 2^2 + 1^2)^{1/2} = \sqrt{14}$

$$\mathbf{I} = \frac{\mathbf{r}}{r} = \frac{3}{\sqrt{14}}\mathbf{i} + \frac{2}{\sqrt{14}}\mathbf{j} + \frac{1}{\sqrt{14}}\mathbf{k}$$

PROJECTION OF $\mathbf{A}$ ON $\mathbf{r}$ IS GIVEN BY $\mathbf{A}\cdot\mathbf{I} = A_xI_x + A_yI_y = A_zI_z$

$$\mathbf{A}\cdot\mathbf{I} = 30\left(\frac{3}{\sqrt{14}}\right) + 50\left(\frac{2}{\sqrt{14}}\right) + 70\left(\frac{1}{\sqrt{14}}\right) = 69.49$$

VECTOR PRODUCT - NOT COMMUTATIVE $\mathbf{A}\times\mathbf{B} \neq \mathbf{B}\times\mathbf{A}$

MOMENT ABOUT A POINT $\mathbf{M} = \mathbf{r}\times\mathbf{F}$

$\mathbf{r}$ IS A VECTOR FROM THE POINT TO THE FORCE.

EXAMPLE: FORCE $\mathbf{F} = 10\mathbf{i} - 10\mathbf{j} + 20\mathbf{k}$ passes through point P(4,2,-2)

CALCULATE MOMENT CAUSED BY $\mathbf{F}$ ABOUT POINT A (2,3,-1)

SOLUTION: $\mathbf{r} = (4-2)\mathbf{i} + (2-3)\mathbf{j} + (-2+1)\mathbf{k} = 2\mathbf{i} - 1\mathbf{j} - 1\mathbf{k}$.

$$\mathbf{M}_A = \mathbf{r}\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & -1 \\ 10 & -10 & 20 \end{vmatrix} = M_x\mathbf{i} + M_y\mathbf{j} + M_z\mathbf{k}$$

$M_x = yF_z - zF_y = (-1)(20) - (-1)(-10) = -30$ k-Ft

$M_y = zF_x - xF_z = (-1)(10) - (2)(20) = -50$ k-Ft

$M_z = xF_y - yF_x = (2)(-10) - (-1)(10) = -10$ k-Ft

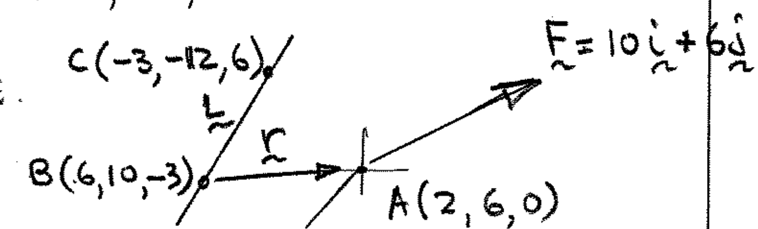
$\mathbf{M}_A = -30\mathbf{i} - 50\mathbf{j} - 10\mathbf{k}$ k-Ft

MOMENT ABOUT AN AXIS (A LINE). - 3 STEPS INVOLVED

FORCE $\underset{\sim}{F} = 10\underset{\sim}{i} + 6\underset{\sim}{j}$ PASSES THROUGH POINT A (2, 6, 0)

FIND MOMENT CAUSED BY $\underset{\sim}{F}$ ABOUT A LINE PASSING THROUGH B (6, 10, -3) AND C (-3, -12, 6)

STEP 1: CALCULATE MOMENT ABOUT A POINT ON THE LINE.



CHOOSE POINT B.
$\underset{\sim}{r}$ IS VECTOR FROM POINT B TO THE FORCE.

$$\underset{\sim}{r} = (2-6)\underset{\sim}{i} + (6-10)\underset{\sim}{j} + (0+3)\underset{\sim}{k} \;;\quad \underset{\sim}{r} = -4\underset{\sim}{i} - 4\underset{\sim}{j} + 3\underset{\sim}{k}$$

$$M_x = yF_z - zF_y = (-4)(0) - (3)(6) = -18$$

$$M_y = zF_x - xF_z = (3)(10) - (-4)(0) = 30$$

$$M_z = xF_y - yF_x = (-4)(6) - (-4)(10) = 16 \qquad \underset{\sim}{M_B} = -18\underset{\sim}{i} + 30\underset{\sim}{j} + 16\underset{\sim}{k}$$

STEP 2: FIND UNIT VECTOR ON AXIS
LET $\underset{\sim}{L}$ BE VECTOR FROM B TO A.

$$\underset{\sim}{L} = (-3-6)\underset{\sim}{i} + (-12-10)\underset{\sim}{j} + (6+3)\underset{\sim}{k}$$

$$\underset{\sim}{L} = -9\underset{\sim}{i} - 22\underset{\sim}{j} + 9\underset{\sim}{k}$$

$$L = \sqrt{9^2 + 22^2 + 9^2} = 25.42$$

UNIT VECTOR $$\underset{\sim}{I} = \frac{\underset{\sim}{L}}{L} = -\frac{9}{25.42}\underset{\sim}{i} - \frac{22}{25.42}\underset{\sim}{j} + \frac{9}{25.42}\underset{\sim}{k}$$

$$\underset{\sim}{I} = -0.354\underset{\sim}{i} - 0.866\underset{\sim}{j} + 0.354\underset{\sim}{k}$$

STEP 3: PROJECT $\underset{\sim}{M_B}$ on $\underset{\sim}{I}$.

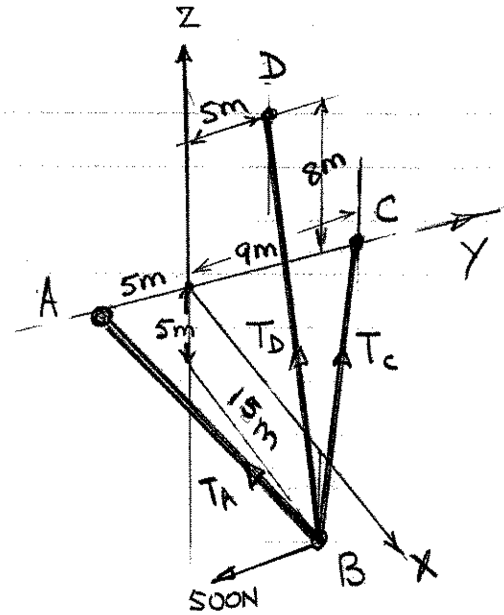
$$M = \underset{\sim}{M_B} \bullet \underset{\sim}{I} = M_x I_x + M_y I_y + M_z I_z$$

$$M = (-18)(-0.354) + (30)(-0.866) + (16)(0.354) = 13.98 \text{ N-m}$$

OR USE DETERMINANT $$M = \underset{\sim}{I} \cdot (\underset{\sim}{r} \times \underset{\sim}{F}) = \begin{vmatrix} 0.354 & -0.866 & 0.354 \\ -4 & -4 & 3 \\ 10 & 6 & 0 \end{vmatrix}$$

$$M = -0.354(-1)^{1+1}\begin{vmatrix} 4 & 3 \\ 6 & 0 \end{vmatrix} + (-0.866)(-1)^{1+2}\begin{vmatrix} -4 & 3 \\ 10 & 0 \end{vmatrix} + 0.354(-1)^{1+3}\begin{vmatrix} -4 & -4 \\ 10 & 6 \end{vmatrix} = 13.98$$

EXAMPLE: Ball joint at A.
Calculate T_D and T_C



REMEMBER TIP MINUS TOE.

FOR T_D: $dx = -15$; $dy = 5$; $dz = 8+5 = 13$

$$d^2 = dx^2 + dy^2 + dz^2 = 15^2 + 5^2 + 13^2$$

$$d = 20.47$$

$$\underset{\sim}{T_D} = T_D \left[\frac{dx}{d} \underset{\sim}{i} + \frac{dy}{d} \underset{\sim}{j} + \frac{dz}{d} \underset{\sim}{k} \right]$$

$$\underset{\sim}{T_D} = \frac{T_D}{20.47} \left(-15\underset{\sim}{i} + 5\underset{\sim}{j} + 13\underset{\sim}{k} \right)$$

$$\underset{\sim}{T_D} = \left(-0.733\underset{\sim}{i} + 0.244\underset{\sim}{j} + 0.635\underset{\sim}{k} \right) T_D$$

Similarly $\underset{\sim}{T_C} = T_C \dfrac{\left(-15\underset{\sim}{i} + 9\underset{\sim}{j} + 5\underset{\sim}{k} \right)}{\sqrt{15^2 + 9^2 + 5^2}} = T_C \left(-0.824\underset{\sim}{i} + 0.495\underset{\sim}{j} + 0.275\underset{\sim}{k} \right)$

Force $\underset{\sim}{F} = -500\underset{\sim}{j}$

NOTE: Force $\underset{\sim}{T_A}$ passes through point A — causes no moment at pt A.

$\Sigma M_A = 0$: $\quad r_{AB}$ is vector from point A to point B.

$$\underset{\sim}{r_{AB}} = 15\underset{\sim}{i} + 5\underset{\sim}{j} - 5\underset{\sim}{k}$$

Due to T_C : $M_x = yF_z - zF_y = 5(0.275T_C) - (-5)(0.495T_C) = 3.85T_C$

$M_y = zF_x - xF_z = -5(-0.824) - 15(0.275) = 0$

$M_z = xF_y - yF_x = (15)(0.495) - (5)(-0.824) = 11.55T_C$

Due to T_D : $M_x = 5(0.635T_D) - (-5)(0.244T_D) = 4.395T_D$

$M_y = -5(-0.733T_D) - 15(0.635T_D) = -5.865T_D$

$M_z = 15(0.244T_D) - 5(-0.733T_D) = 7.325T_D$

Due to 500 N $\quad M_x = 0 - (-5)(-500) = -2500$

$M_y = 0$

$M_z = 15(-500) = -7500$

$\Sigma M_x = 0: \quad 3.85 T_C + 4.395 T_D - 2500 = 0 \quad (1)$

$\Sigma M_y = 0 \quad 0 - 5.865 T_D = 0 \quad (2)$

$\Sigma M_z = 0 \quad 11.55 T_C + 7.325 T_D - 7500 = 0 \quad (3)$

From (3) $\quad T_D = 0$

From (1) $\quad T_C = 649$ N

From (3) $\quad T_C = 649$ N

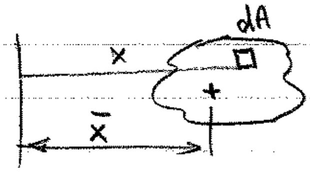
SOLVE PROBLEMS 4.64, 4.70, 4.75, 4.92, 4.102

CHAPTER 5 - CENTROIDS

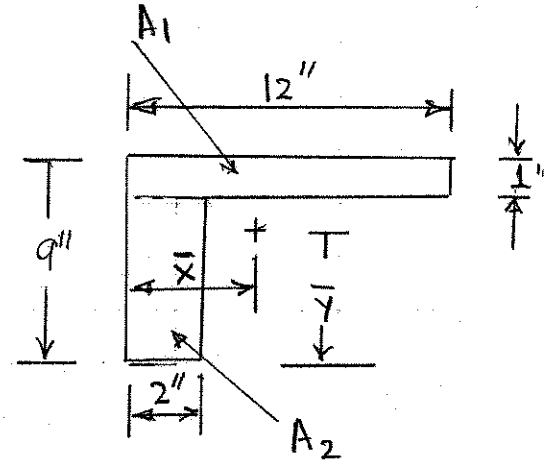
1. Moment caused by $dA = x \cdot dA$

Sum $= \Sigma x\,dA = (\Sigma dA)\,\bar{x}$

$$\bar{x} = \frac{\Sigma x\,dA}{\Sigma dA} = \frac{\Sigma x_i A_i}{A_i}$$



$$\bar{y} = \frac{\Sigma A_i y_i}{A_i}$$



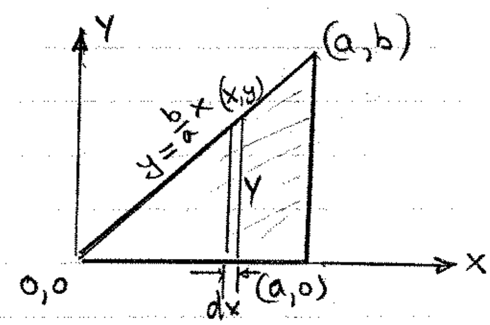
2.

Area	A (in^2)	X (in)	Y (in)	Ax (in^3)	Ay
A_1	12	6	8.5	72	102
A_2	16	1	4	16	64
	$\Sigma A = 28$			$\Sigma Ax = 88$	$\Sigma Ay = 166$

$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} = \frac{88}{28} = 3.143''$$

$$\bar{Y} = \frac{\Sigma AY}{\Sigma A} = \frac{166}{28} = 5.929''$$

2. CENTROID BY INTEGRATION



$dA = y\,dx$

$$\bar{x} = \frac{\Sigma x\,dA}{\Sigma dA} = \frac{\int x\,dA}{\int dA}$$

$$\int x\,dA = \int xy\,dx = \int_0^a x\left(\frac{b}{a}x\right)dx$$

$$= \frac{b}{a}\left[\frac{x^3}{3}\right]_0^a = \frac{ba^2}{3}$$

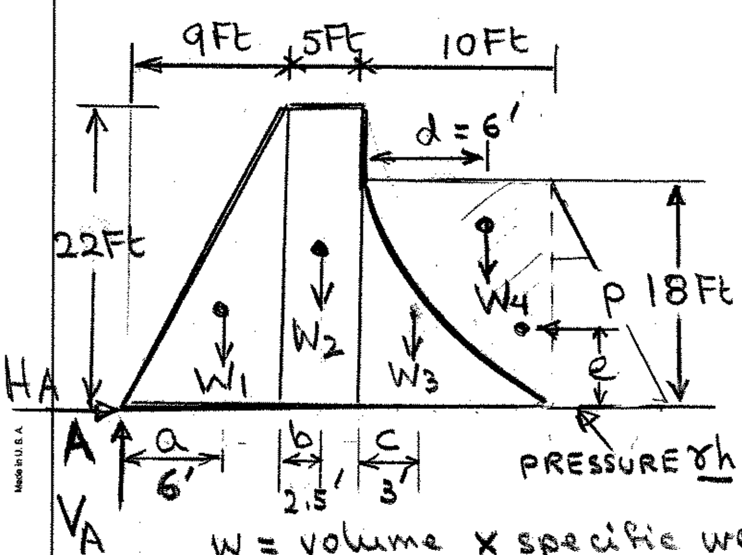
$$\int dA = \int y\,dx = \int_0^a \frac{b}{a}x\,dx = \frac{b}{a}\frac{x^2}{2}\Big]_0^a = \frac{ab}{2}$$

$$\therefore\ \bar{x} = \frac{ba^2}{3}\Big/\frac{ab}{2} = \frac{2a}{3}$$

DO

3. SAMPLE PROBLEM 5.10 P251

FORCES ON CONCRETE DAM



WORK WITH 1-FT-THICK SECTION

$w_c = 150\ lb/ft^3$ NOTE UNITS

$\gamma_w = 62.4\ lb/ft^3$ SPECIFIC WEIGHT

W_1 = Wt OF TRIANGULAR SECTION
W_2 = Wt OF RECTANGULAR SECTION
W_3 = Wt OF PARABOLIC SECTION
W_4 = Wt OF WATER

W = volume x specific weight   VOLUME = AREA x THICKNES

$W_1 = \frac{1}{2}(9)(22)(1) \times 150 = 14,850\ lb$

$W_2 = (5)(22)(1) \times 150 = 16,500\ lb$

$W_3 = (\frac{1}{3})(10)(18)(1) \times 150 = 9,000\ lb$

$\boxed{W_4 = \frac{2}{3}(10 \times 18 \times 1)\,62.4 = 748 \quad P = \frac{(\gamma h)}{2}(h \times 1) = \frac{1}{2}(62.4)(18 \quad P = 10,109\ lb}$

Location of W_1 : $a = \frac{2}{3}(9) = 6\ Ft$ TRIANGLE

Location of W_2 : $b = \frac{1}{2}(5) = 2.5\ Ft$ RECTANGLE

Location of W_3 : $c = \frac{3h}{10} = \frac{3}{10}(10) = 3\ Ft$ PARABOLA   P225

Location of W_4 : $d = \frac{3h}{5} = \frac{3}{5}(10) = 6\ Ft$ PARABOLA   P225

Location of P : $e = \frac{1}{3}(18) = 6\ Ft$ TRIANGLE.

ASSUME DAM WILL ROTATE ABOUT POINT A. (LIFTING)

$\Sigma F_x = 0$: $H_A - P = 0$; $H_A = P = 10,109\ lb.$

$\Sigma F_y = 0$: $V_A - W_1 - W_2 - W_3 - W_4 = 0$; $V_A = W_1 + W_2 + W_3 + W_4 = 47836\ lb$

OVERTURNING MOMENT : $Pe = 10,109(6) = 60,654\ lb\text{-}ft.$

RESTORING MOMENT : $W_1 a + W_2(9+b) + W_3(9+5+c) + W_4(9+5+d)$

$= 14,850(6) + 16,500(11.5) + 9,000(17) + 7488(20)$

$= 581,610\ lb\text{-}ft.$

RESTORING MOMENT > OVERTURNING MOMENT   O.K.