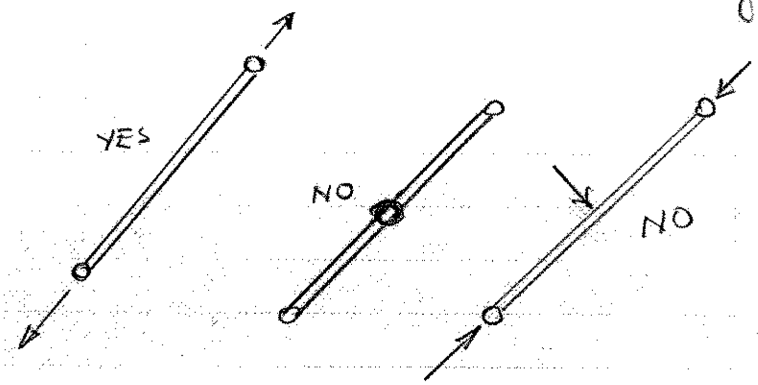
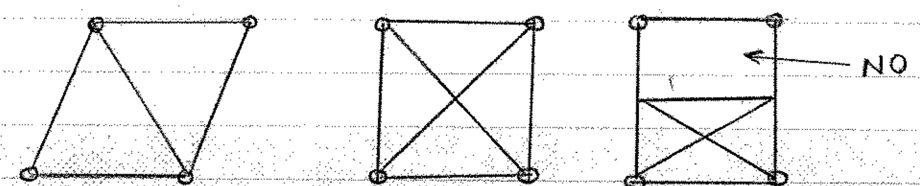
# CHAPTER 6 - TRUSSES

1. A truss member : (a) starts at a pinned joint and ends at a pin joint.
   (b) is not continuous through a joint.
   (c) has no load applied directly on it.

As a result of (a), (b) & (c) carries axial load only. (Tension or compression)




2. Stable trusses - All panels must be triangulated. Reactions stable

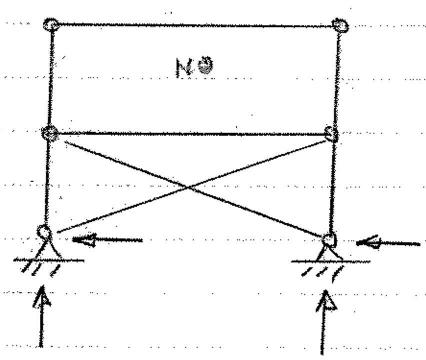



STABLE STABLE UNSTABLE

3. m = number of members ; r = number of reactions ; j = number of joints.

[2D] trusses: Determinate if $m+r = 2j$. ⎫ must have all panels triangulated
Indeterminate if $m+r > 2j$. ⎭
Unstable if $m+r < 2j$

<u>CAREFUL</u>




$m = 8$ ; $r = 4$ ; $j = 6$

$m+r = 8+4 = 12$

$2j = 2(6) = 12$

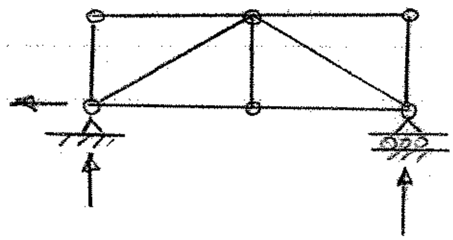
$m+r = 2j$ appears determinate.

ACTUALLY UNSTABLE.

→ <u>SIGN CONVENTION</u> TENSION → POINTING AWAY FROM JOINT.

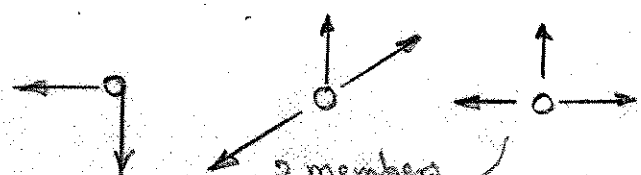
## TRUSSES

1. Supports must be adequate. Three reactions or more.



3 equations of equilibrium.
$\Sigma F_x = 0$ ; $\Sigma F_y = 0$ ; $\Sigma M = 0$.

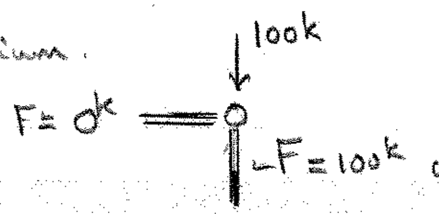
2. <u>METHOD OF INSPECTION</u>

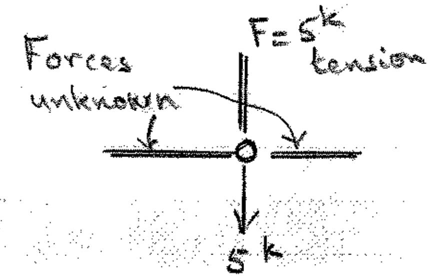


2 members at right angles

2 members parallel

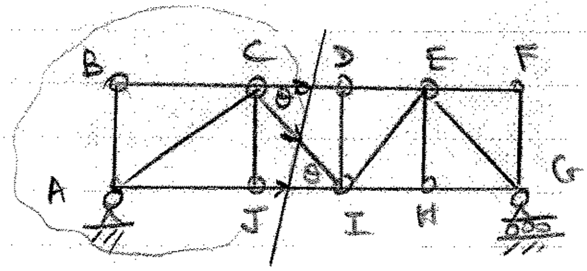
Joint equilibrium.

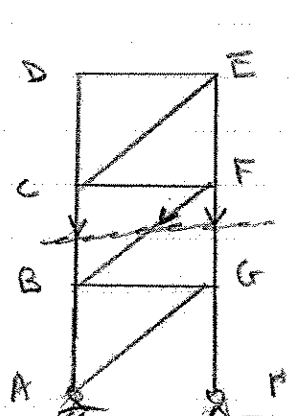


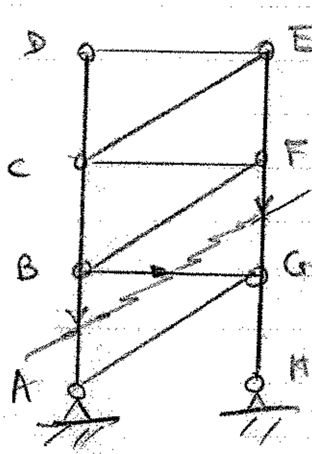




3. <u>METHOD OF SECTION</u>

Cut through 3 members at most
CONSIDER LEFT OR RIGHT SIDE.
To find $F_{CD}$, $\Sigma M_I = 0$
To find $F_{JI}$, $\Sigma M_C = 0$
To find $F_{CI}$, $\Sigma F_y = 0$



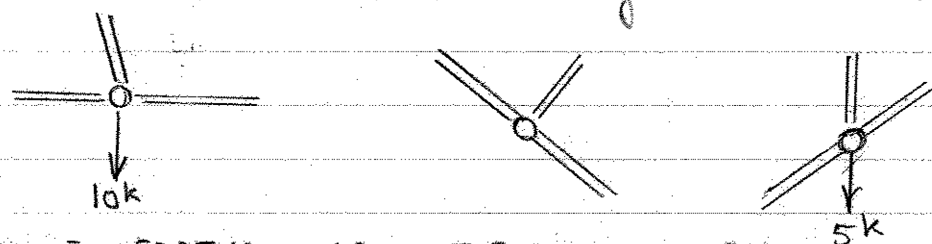



4. 3 METHODS ; (a) INSPECTION (b) SECTION (c) JOINTS

5 METHOD OF INSPECTION .

Look for joints where two members are collinear and concurrent, and where there are only 3 members. $\boxed{\text{2 members at rt } \angle\text{s}}$




RESOLVE PERPENDICULAR TO THE TWO COLLINEAR MEMBERS.

→ Example:- P 299 Pr 6.3 ; 6.6 ; 6.15 ; 6.17 ; 6.22 ; 6.23 ; 6.26 ; 6.30

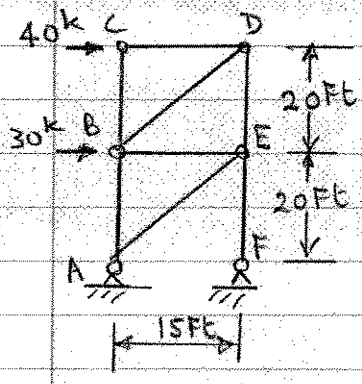
6. METHOD OF SECTION

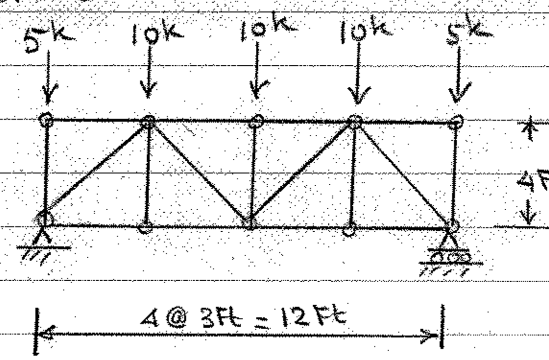
(a) Horizontal trusses → Find reactions

(b) Vertical trusses → no need to find reactions.

(c) Cut not more than 3 members.

(d) Use $\Sigma F_x = 0$ ; $\Sigma F_y = 0$ ; $\Sigma M = 0$




HOW MANY REACTIONS AT F ??

7. METHOD OF JOINTS

Move from joint to joint. Cannot have more than 2 UNKNOWN.

# Simple Frames

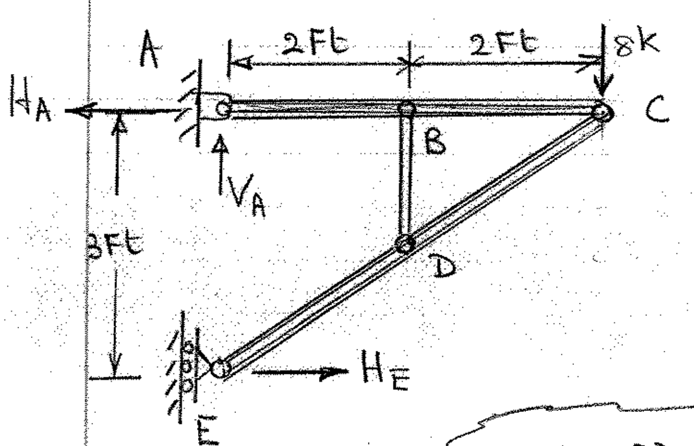
1. METHOD:

(a) CALCULATE REACTIONS

(b) LOOK FOR TRUSS MEMBERS

(c) SEPARATE MEMBERS

(d) WORK WITH DETERMINATE MEMBERS FIRST.




SOLUTION:

$\Sigma M_A = 0: \quad -8(4) + H_E(3) = 0$

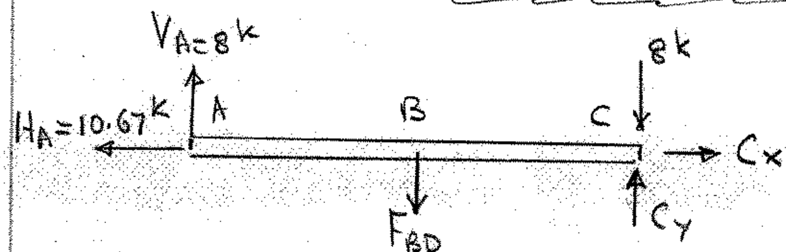
$H_E = 10.67\,k$

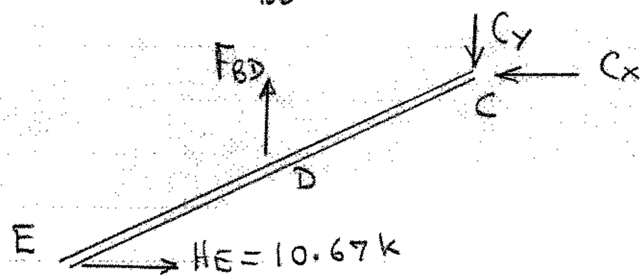
$\Sigma F_x = 0: \quad H_E - H_A = 0$

$H_A = 10.67^k$

$\Sigma F_y = 0: \quad V_A = 8^k$

NOTE: BD IS A TRUSS MEMBER



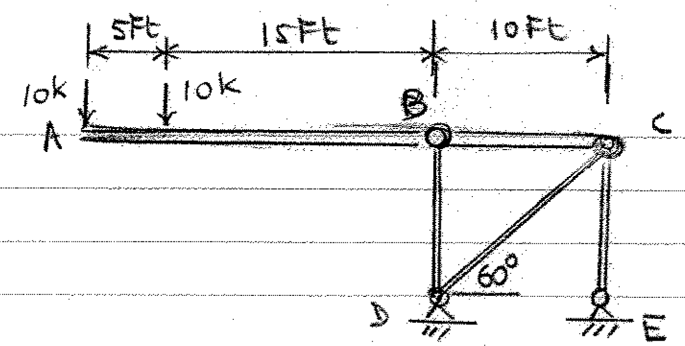

Work with EBC: $\quad \Sigma M_c = 0: \quad -F_{BD}(2) + 10.67(3) = 0$

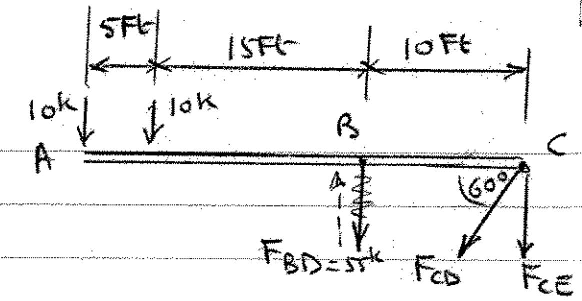
$F_{BD} = 16\,k$

$\Sigma F_x = 0: \quad H_E - C_x = 0 \qquad C_x = 10.67^k$

$\Sigma F_y = 0: \quad F_{BD} - C_y = 0 \qquad C_y = 16\,k$

2.




BD, DE and CE ARE TRUSS MEMBERS

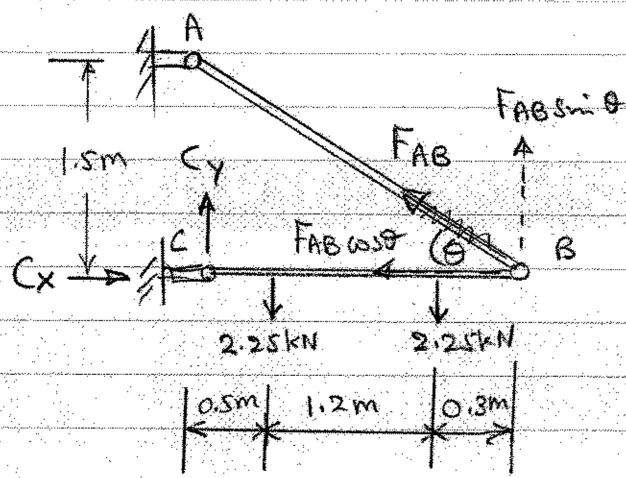
$\Sigma M_C = 0: \quad 10(30) + 10(25) + F_{BD}(10) = 0$

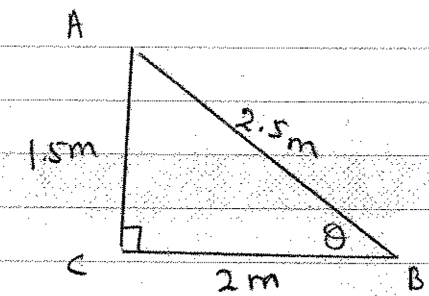
$F_{BD} = -55k$

WHY NOT $\Sigma F_y = 0$ ?? $\Sigma F_x = 0: \quad -F_{CD}\cos 60° = 0 \rightarrow F_{CD} = 0$

$\Sigma F_y = 0: \quad -10 - 10 + 55 - F_{CE} = 0 \qquad F_{CE} = 35^k$

3. EXAMPLE




AB is a truss member

Resolve $F_{AB}$ into 2 components

$\Sigma M_C = 0: \quad F_{AB}\sin\theta\,(2) - 2.25(0.5) - 2.25(1.7) = 0$

$F_{AB}\left(\frac{1.5}{2.5}\right)(2) = 4.95$

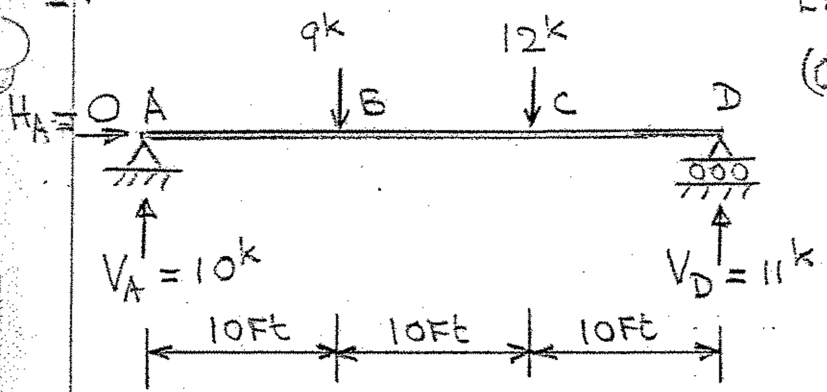
$F_{AB} = \frac{4.95(2.5)}{(1.5)2} = 4.125\text{ kN}$

$\Sigma F_x = 0: \quad C_x - F_{AB}\cos\theta = 0 \qquad C_x = 4.125\left(\frac{2}{2.5}\right) = 3.3\text{ kN}$

$\Sigma F_y = 0: \quad C_y + F_{AB}\sin\theta - 2.25 - 2.25 = 0$

$C_y = 4.5 - 4.125\left(\frac{1.5}{2.5}\right) = 2.025\text{ kN}$

1.




EXERCISE: Calculate $V_{c,R}$ & $BM_c$.

(a). Calculate Reactions

$\Sigma M = 0$ FIRST

$\Sigma M_A = 0: -9(10) - 12(20) + V_D(30) = 0$

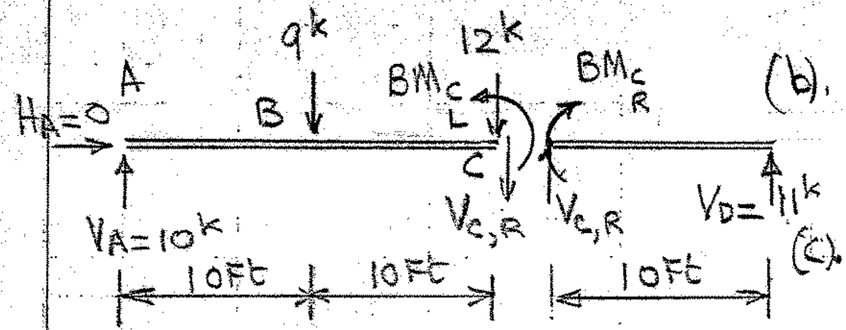
$V_D = 11^k$

Then $\Sigma F_x = 0$ and $\Sigma F_y = 0$

$\Sigma F_x = 0: H_A = 0$

$\Sigma F_y = 0: V_A - 9 - 12 + 11 = 0$

$V_A = 10^k$




(b). Put numerical values in free-body diagram. (FBD)

(c). LONG METHOD: Make cut. Draw FBD. Use equilibrium principle.

$\Sigma F_y = 0: 10 - 9 - 12 - V_{c,R} = 0$

$V_{c,R} = 10 - 9 - 12 = -11^k$

$\Sigma M_c = 0: -10(20) + 9(10) + BM_{c_L} = 0$

$BM_{c_L} = 10(20) - 9(10) = 110^{k\text{-}ft}$

## SHORT METHOD

SHEAR : Start from left. Upward force positive. Downward force negative.

$V_{c,R} = 10 - 9 - 12 = -11^k$ (same as above)

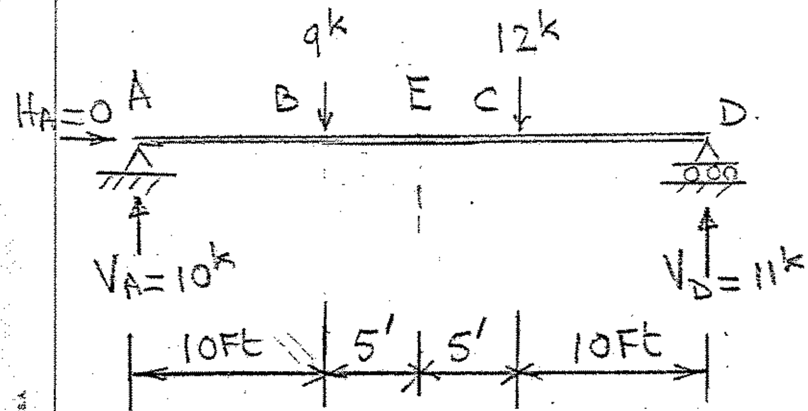
BM : Left arm up is positive (starting from left)

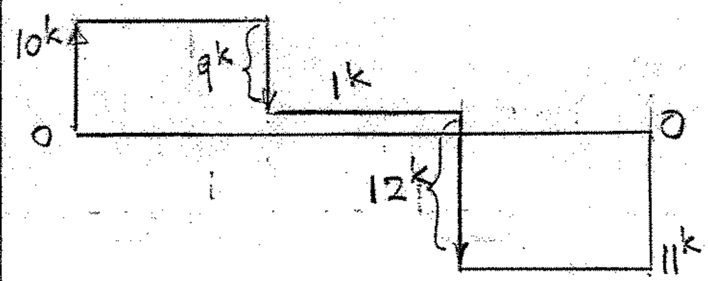
$BM_{c_L} = \overset{up}{10(20)} - \overset{down}{9(10)} = 110^{k\text{-}ft}$ (same as above).

BM : Right arm up is positive, starting from the right.

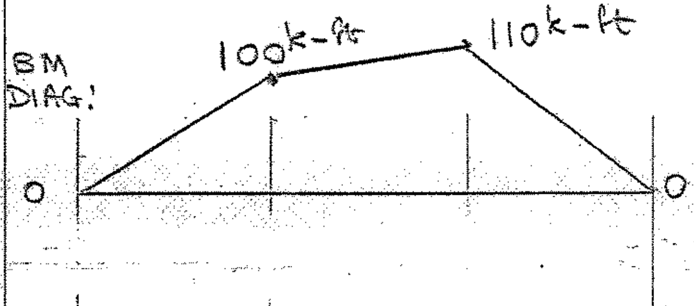
$BM_{c_R} = \overset{up}{11(10)} = 110^{k\text{-}ft}$ SINCE $BM_{c_L} = BM_{c_R}$, [illegible]

2. Shear and BM diagrams




V – diagram




$BM_{A_L} = 0$ (no forces left of A)

$BM_{D_R} = 0$ (no forces right of D)

$BM_{B_L} = 10(10) = 100^{k\text{-}ft}$ (left arm)

Check: $BM_{B_R} = 11(20) - 12(10) = 100^{k\text{-}ft}$ (right arm)

$BM_{C_L} = 10(20) - 9(10) = 110^{k\text{-}ft}$

$BM_{C_R} = 11(10) = 110^{k\text{-}ft}$.

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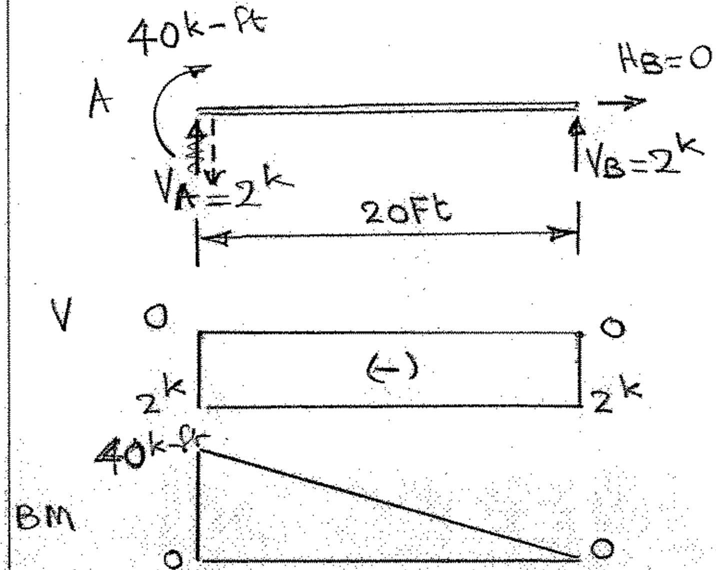
CALCULATE BM at point E.

$BM_{E_L} = 10(15) - 9(5) = 105^{k\text{-}ft}$

OR $BM_{E_R} = 11(15) - 12(5) = 105^{k\text{-}ft}$

3\. CONCENTRATED MOMENTS




Calculate Reactions.

$H_B = 0$ $\quad \Sigma M_B = 0$:

$$-40 - V_A(20) = 0$$

$$V_A = -2^k$$

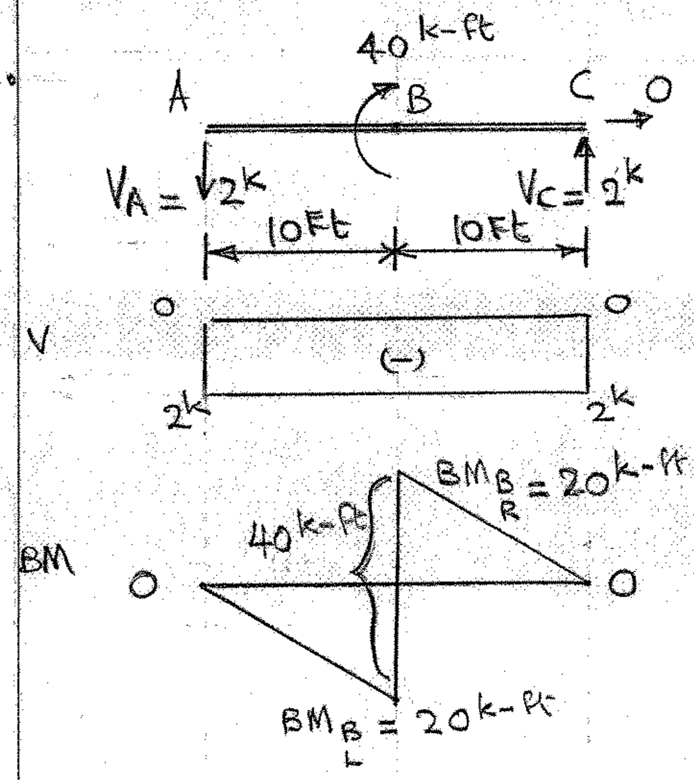
$\Sigma F_y = 0$:

$$V_A + V_B = 0$$

$$V_B = -V_A = +2^k$$

NOW TRANSFER NUMBERS TO FBD
DRAW V & BM DIAGRAMS

4\.



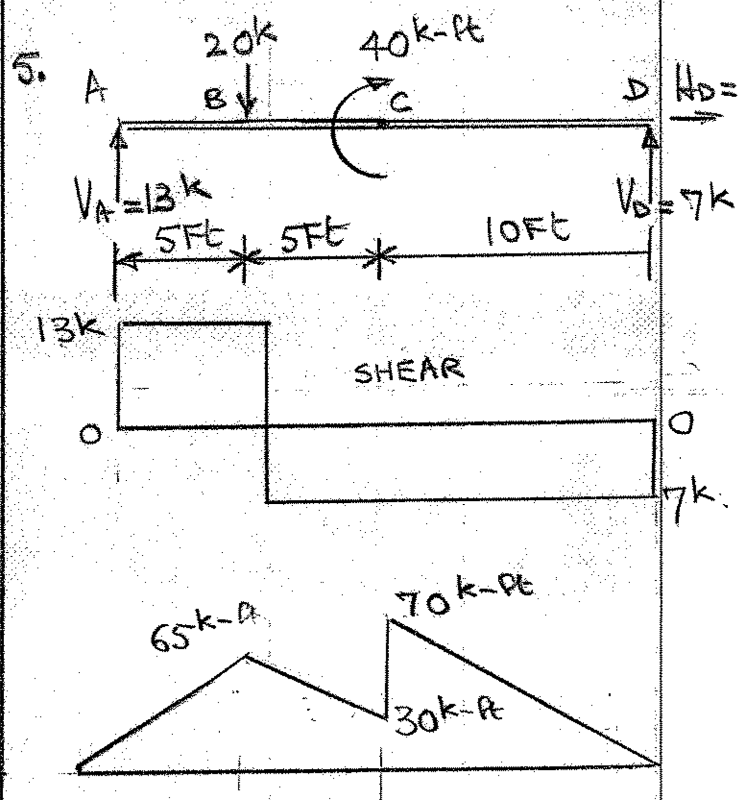

TO THE LEFT OF B:

$$BM_{B_L} = -2(10) = -20^{k-ft}$$

TO THE RIGHT OF B:

$$BM_{B_R} = 2(10) = +20^{k-ft}$$

5\.



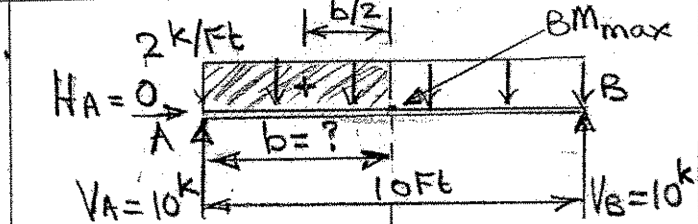

$$BM_{B_L} = 13(5) = 65^{k-ft}$$

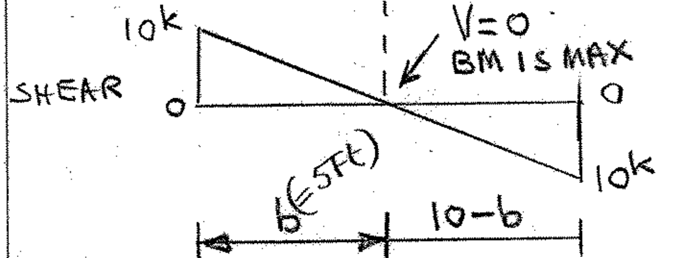
$$BM_{C_L} = 13(10) - 20(5) = 30^{k-ft}$$

$$BM_{C_R} = 7(10) = 70^{k-ft}$$

## UNIFORM LOAD

6.




Locate where V = 0.

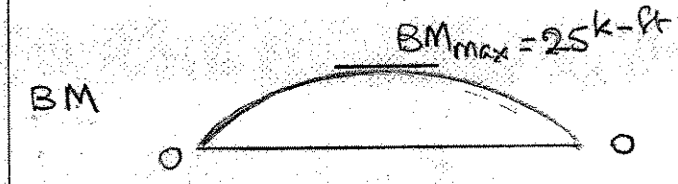
Similar triangles:

$$\frac{10}{b} = \frac{10}{10-b}$$

$$b = 5\text{Ft}$$

$$BM_{max} = 10(5) - (2\times 5)\frac{5}{2}$$

$$= 25^{k\text{-}ft}$$

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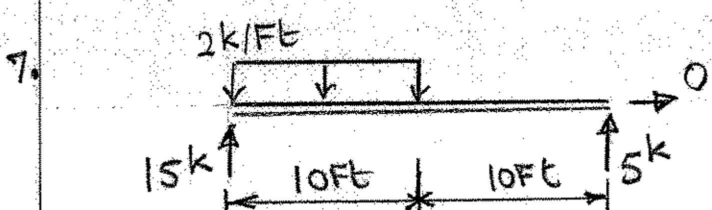
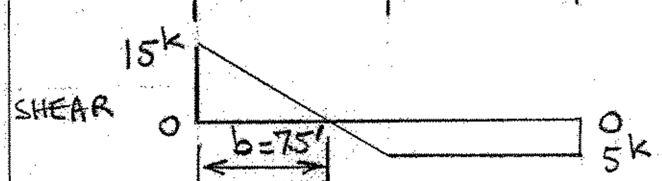
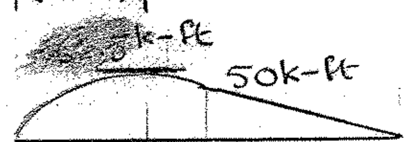
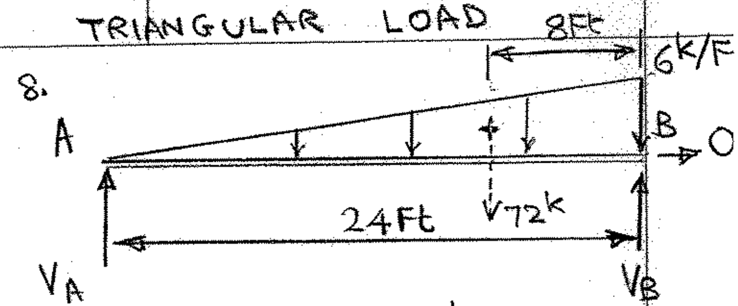



## TRIANGULAR LOAD

8.



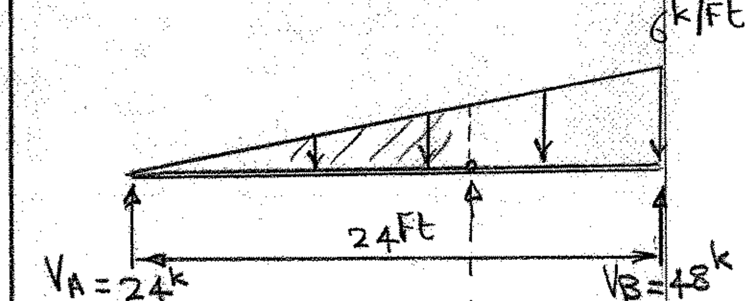

$$\text{Area} = \frac{1}{2}(24)(6) = 72^k$$

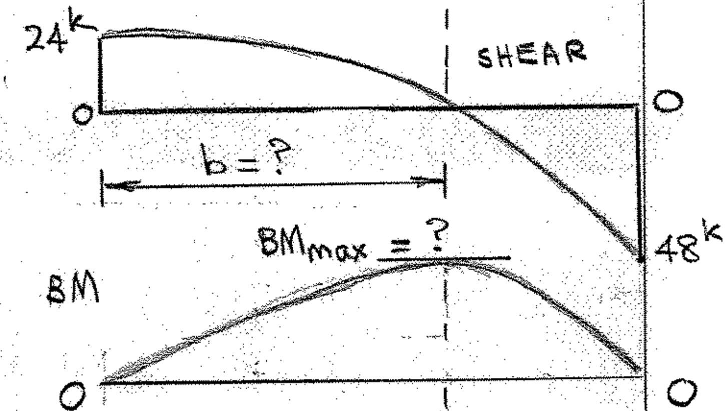
$$\Sigma M_B = 0: -V_A(24) + 72(8) = 0$$

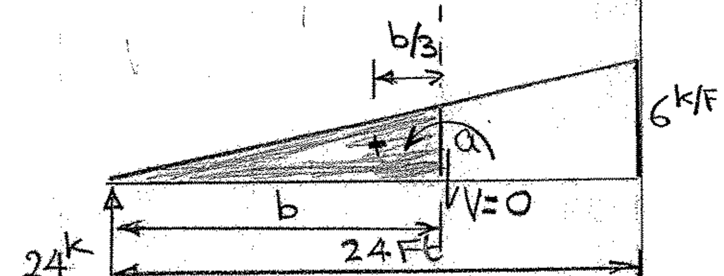
$$V_A = 24^k$$

$$\Sigma F_y = 0: \quad V_A + V_B - 72 = 0$$

$$V_B = 48^k$$




$$\text{SIMILAR } \Delta\text{'s}: \quad \frac{a}{b} = \frac{6}{24}; \quad a = b/4$$

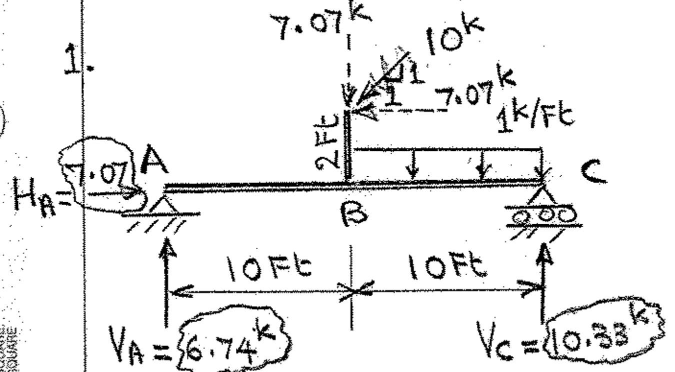
$$V = 0: \quad 24 = \frac{1}{2}b(a) = \frac{1}{2}b\left(\frac{b}{4}\right)$$

$$b = 13.84\text{Ft}$$

$$BM_{max} = 24(13.84) - \underbrace{\frac{1}{2}(b)(a)}_{\text{area}}\underbrace{\left(\frac{b}{3}\right)}_{C.G}$$

$$BM_{max} = 221.7\ k\text{-}ft$$

1.




$\Sigma M_A = 0:$

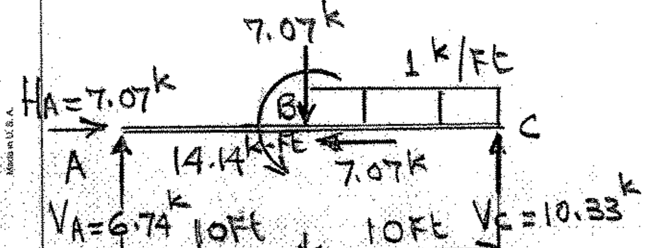
$-7.07(10) + 7.07(2) - (1 \times 10)15 + V_C(20) = 0$

$V_C = 10.33^k$

$\Sigma F_y = 0: \quad V_A + V_C - 7.07 - 10 = 0$

$V_A = 6.74^k$

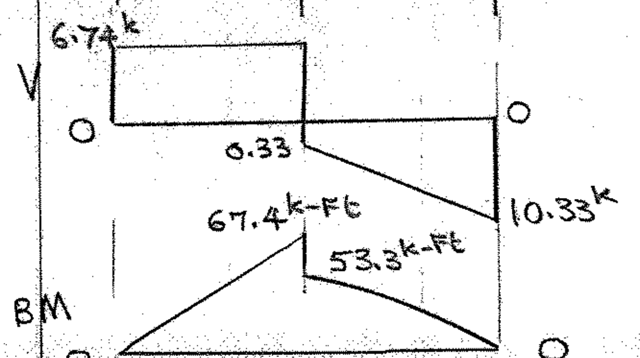
$\Sigma F_x = 0: \quad H_A = 7.07^k$




$BM_{A_L} = 0 \quad ; \quad BM_{C_R} = 0$

$BM_{B_L} = 6.74(10) = 67.4^{k\text{-}Ft}$

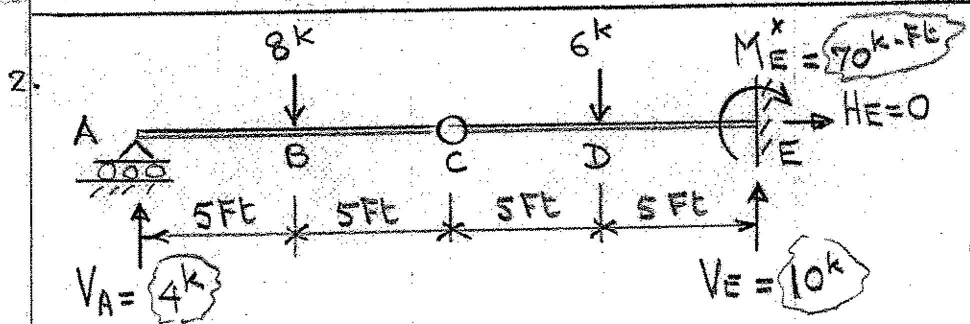
$BM_{B_R} = 10.33(10) - (1 \times 10)\left(\frac{10}{2}\right) = 53.3^{k\text{-}Ft}$




CHECK: $BM_{B_L} - BM_{B_R} = 67.4 - 53.3 = 14.1^{k\text{-}Ft}$

EQUALS CONCENTRATED MOMENT: GOOD

2.




SEGMENT ABC:

$\Sigma M_C = 0:$

$-V_A(10) + 8(5) = 0$

$V_A = 4^k$

ENTIRE BEAM ABCDE

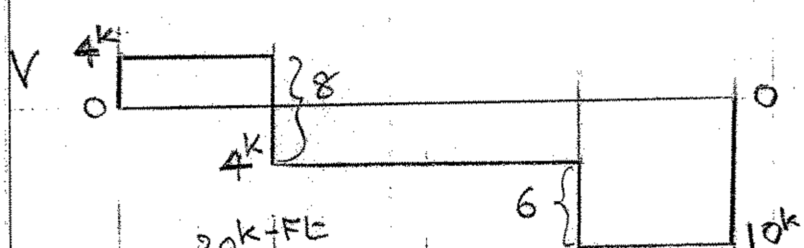
$\Sigma F_y = 0: \; V_A + V_E - 8 - 6 = 0$

$V_E = 10^k$

$\Sigma M_E = 0:$

$-V_A(20) + 8(15) + 6(5) - M_E^* = 0$

$M_E^* = 70^{k\text{-}Ft}$

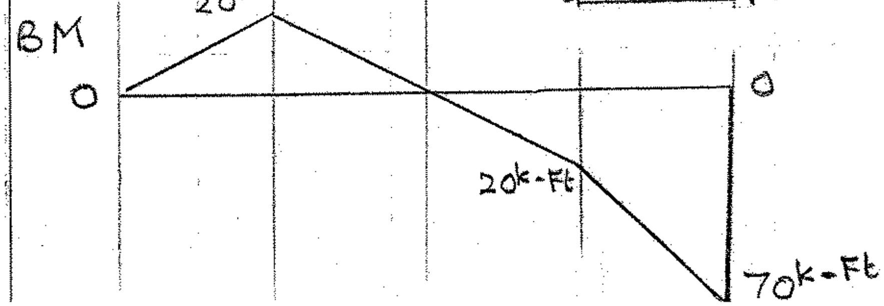



CHECK SEGMENT CDE

$\Sigma M_C = -6(5) - M_E^* + V_D(10)$

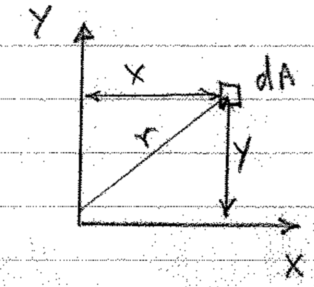
$= -30 - 70 + 10(10) = 0$

GOOD.



# CHAPTER 9 - MOMENT OF INERTIA

1.




$I_x = \int y^2 dA$ DEFINITION

$I_y = \int x^2 dA$

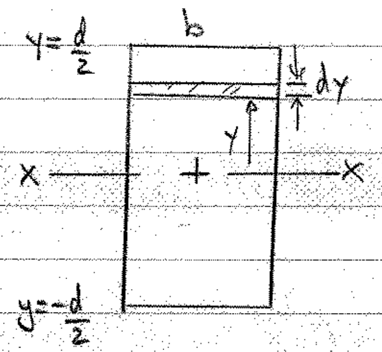
$I_{xy} = \int xy\, dA$

$I_p = \int r^2 dA = \int (x^2+y^2) dA = \int y^2 dA + \int x^2 dA$

POLAR M.I. $I_p = I_x + I_y$

RECTANGLE

2. CENTROIDAL AXIS



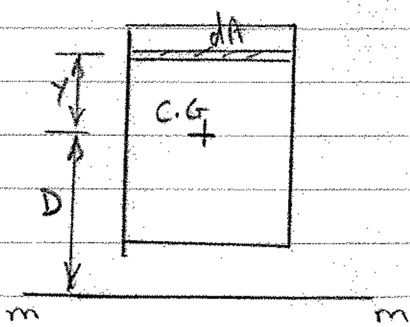

$dA = b \cdot dy$

$I_x = \int y^2 dA = \int_{-d/2}^{d/2} y^2 \cdot b \cdot dy$

$I_x = b \left. \frac{y^3}{3} \right]_{-d/2}^{d/2} = \frac{b}{3}\left[\frac{d^3}{8} + \frac{d^3}{8}\right]$

$\boxed{I_x = \frac{1}{12} b d^3}$ ALWAYS CUBE DIMENSION PERPENDICULAR TO AXIS.

3. PARALLEL AXIS THEOREM




$I_m = \int (y+D)^2 dA = \int (y^2 + 2yD + D^2) dA$

$I_m = \int y^2 dA + \int 2yD\, dA + \int D^2 dA$

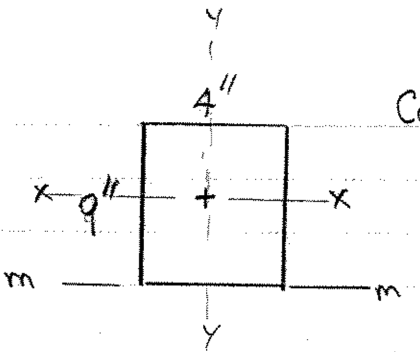
NOTE $\int y^2 dA = I_x$ about centroid.

$D^2$ = constant $\int dA = A$

$y\,dA = 0$ at origin $\bar{y} = \frac{\int y\, dA}{\int dA} = 0$

$\therefore \boxed{I_m = I_x + AD^2}$

4.




Calculate $I_m$.

$I_x = \frac{1}{12}(4)(9)^3 = 243 \text{ in}^4$

$A = 9 \times 4 = 36 \text{ in}^2$

$D = 4.5$

$AD^2 = 36(4.5)^2 = 729 \text{ in}^4$

$I_m = I_x + AD^2 = 243 + 729 = 972 \text{ in}^4$

FORMULA $I_m = \frac{1}{3}bd^3 = \frac{1}{3}(4)(9)^3 = 972 \text{ in}^4$

5. Calculate $I_p$ for above rectangle.

$I_x = \frac{1}{12}(4)(9)^3 = 243 \text{ in}^4$

$I_y = \frac{1}{12}(9)(4)^3 = 48 \text{ in}^4$

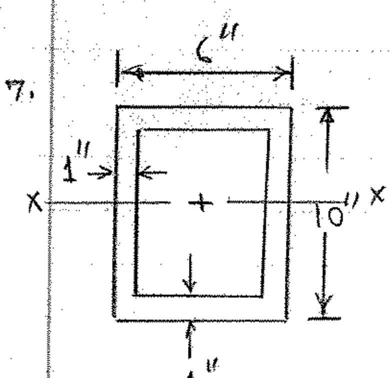
$I_p = I_x + I_y = 243 + 48 = 291 \text{ in}^4$

6. Calculate $r_x$ and $r_y$ $\boxed{I = Ar^2}$ $r = \sqrt{I/A}$

$r_x = \sqrt{I_x/A} = \sqrt{\frac{243}{36}} = 2.6 \text{ in}$

$r_y = \sqrt{I_y/A} = \sqrt{\frac{48}{36}} = 1.15 \text{ in.}$

7.

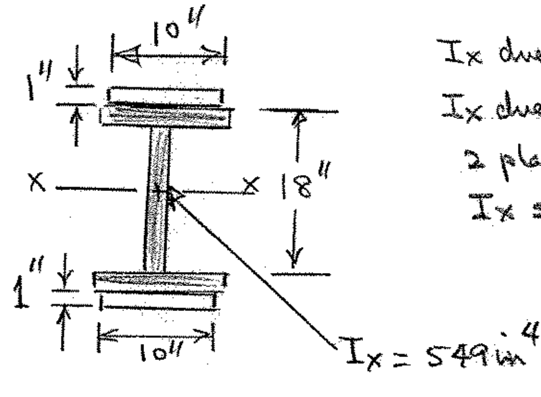



SYMMETRICAL CASE

$I_x = \frac{1}{12}(6)(10)^3 - \frac{1}{12}(4)(8)^3$

$I_x = 500 - 170.67 = 329.83 \text{ in}^4$

8.




$I_x$ due to I-shape $= 549 \text{ in}^4$ (given)

$I_x$ due to 1″x10″ plate $= \frac{1}{12}(10)(1)^3 + (10 \times 1)\,9.5^2 = 903.3 \text{ in}$

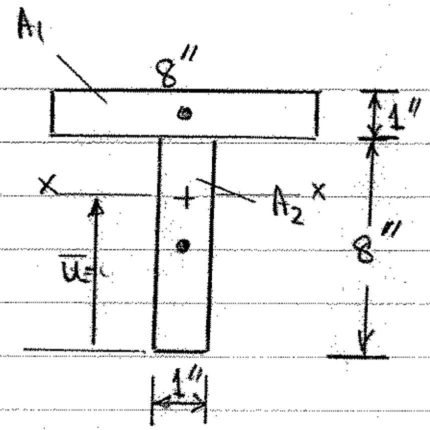
2 plates

$I_x = 549 + 2(903.3) = 2{,}355.7 \text{ in}^4$.

9. SOLVE T-SECTION

LOCATE CENTROID

| Area | $A(in^2)$ | $u(in)$ | $Au(in^3)$ |
|---|---|---|---|
| $A_1$ | 8 | 8.5 | 68 |
| $A_2$ | 8 | 4 | 32 |
| | 16 | | 100 |




$$\bar{u} = \frac{\Sigma(Au)}{\Sigma A} = \frac{100}{16} = 6.25 \text{ in}$$

$$I_x \text{ due to } A_1 = \frac{1}{12}(8)(1)^3 + (8\times1)(8.5-6.25)^2 = 41.17 \text{ in}^4$$

$$I_x \text{ due to } A_2 = \frac{1}{12}(1)(8)^3 + (8\times1)(6.25-4)^2 = 83.17 \text{ in}^4$$

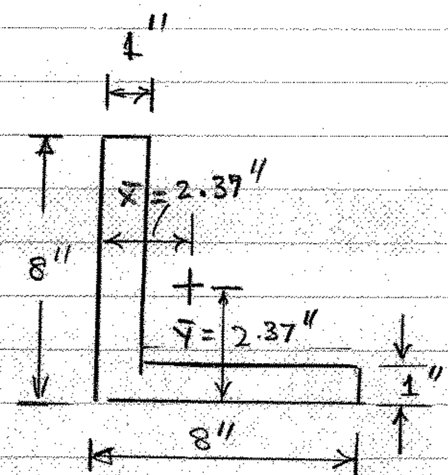
$$\text{Total } I_x = 41.17 + 83.17 = 124.34 \text{ in}^4$$

10. SECTIONS WITH NO AXIS OF SYMMETRY

$$I_{max} = \frac{I_x + I_y}{2} + \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

$$I_{min} = \frac{I_x + I_y}{2} - \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

$$\tan 2\theta = \frac{-2I_{xy}}{I_x - I_y}$$



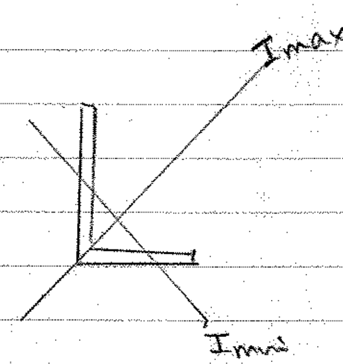

Locate C.G. Already done for you $\bar{x} = 2.37''$; $\bar{y} = 2.37''$

$I_x = 89 \text{ in}^4$

$I_y = 89 \text{ in}^4$

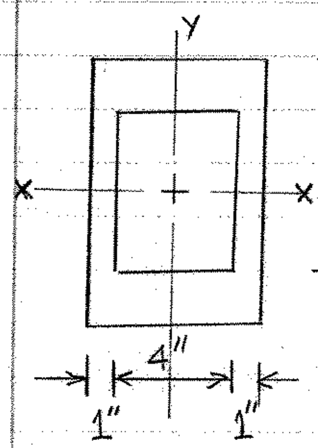
$I_{xy} = A_1 x_1 y_1 + A_2 x_2 y_2$

$$r_{min} = \sqrt{\frac{I_{min}}{A}}.$$

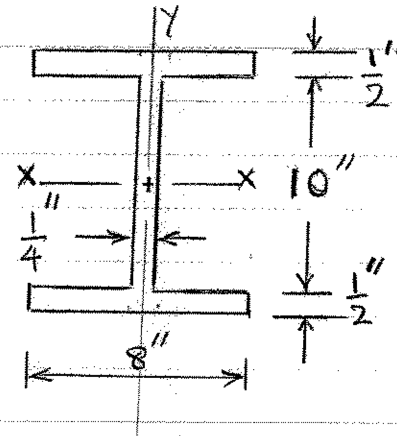


# MOMENT OF INERTIAS – HOMEWORK

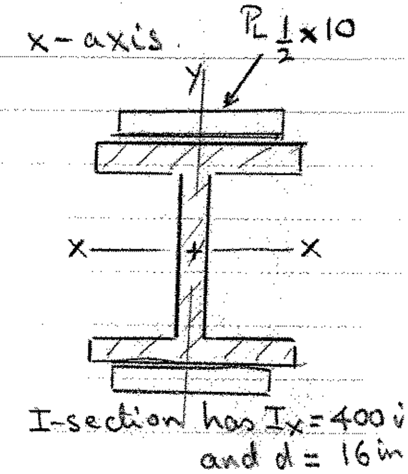
1. Calculate the moment of inertia about the x-axis.



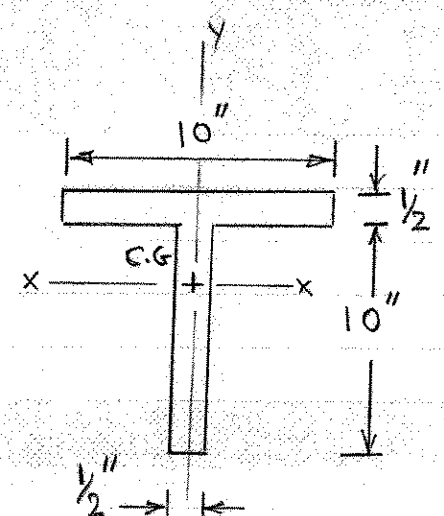

(a)



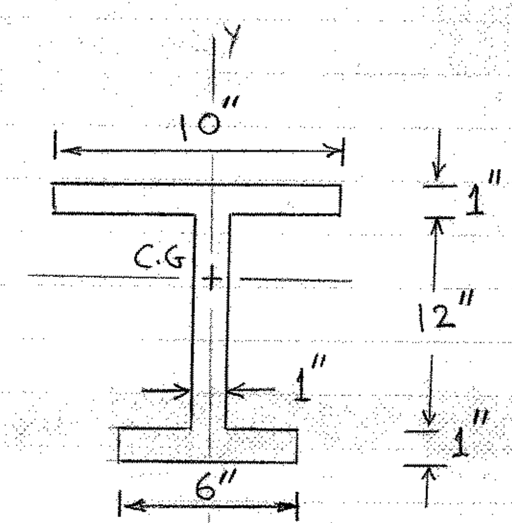

(b)




(c)




(d)




(e)

2. Calculate $I_y$ for above sections.

3. Calculate $r_x$ and $r_y$ for above sections.

4. $I_x$, $I_y$, $I_u$ and $I_v$ can be calculated.
The vertical leg of the angle is longer than the horizontal leg.
(a) List $I_x$, $I_y$, $I_u$ and $I_v$ in order of increasing magnitude

(b) About which axis is the radius of gyration, r, smallest?